CDC 2002 INVITED SESSIONS

1. **Name of Sessions**

   Nonsmooth Analytic Methods in Control Theory I-II

2. **Administrators for Sessions**

   a) Session I:
      Organizer: Michael A. Malisoff
      Co-Organizer: Peter R. Wolenski
      Chair: Peter R. Wolenski
      Co-Chair: Michael A. Malisoff

   b) Session II:
      Organizer: Michael A. Malisoff
      Co-Organizer: Peter R. Wolenski
      Chair: Peter R. Wolenski
      Co-Chair: Michael A. Malisoff

3. **Contact Information for Organizer and Chair of Sessions**

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4. **Topics and Purpose of the Sessions**

   Engineering and Applied Mathematics abound with challenging situations where one seeks to control the behavior of trajectories of dynamical systems. For example, it is often desirable to bring the state of a dynamical system toward a given set, to keep the state in a set, or to find trajectories that optimize a given criterion. These considerations are coupled with related issues involving the construction of feedback control mechanisms to achieve the desired aims.

   It has become clear over the past two decades that classical mathematical concepts are not wholly adequate to describe and prescribe the behavior of controlled dynamical systems, largely because nonsmoothness arises naturally when min operations are present or nonlinear systems develop shocks. The emergent theory of nonsmooth or variational analysis has been developed in part to supply the requisite mathematical tools needed to systematically handle these phenomena. Combined with more traditional control methods, new powerful techniques based on nonsmooth analysis have been developed to analyze and control trajectories of dynamical systems.

The purpose of these sessions is to bring together prominent international researchers that work on important new aspects of controlled dynamical systems that require nonsmooth techniques. The two sessions form a cohesive, well integrated exploration of those areas of mathematical control theory in which nonsmooth analysis has had a major impact. The sessions will complement the sessions "Geometric Methods In Control Theory I-II: Special Sessions Dedicated To Jack Warga's 80th Birthday" and "Variational Analysis and Optimal Control I-III: Special Sessions Dedicated To Jack Warga's 80th Birthday" which were also proposed for this conference. The first session will center on Hamilton-Jacobi equations and inequalities. The second session will present new results in set-valued differentials, and will illustrate how generalized differentials can be used in diverse control applications, such as impulsive and hybrid system theory, necessary conditions for optimality, and stability radii for exponential stabilization. All the speakers in the two sessions are familiar with the motivation and required background material to appreciate every talk.

5. **Session I: Hamilton-Jacobi Equations and Inequalities**

The theory of Hamilton-Jacobi (HJ) equations and inequalities arises in a large variety of important settings in control theory and other areas of applied mathematics. Probably most important to control is the fact that Lyapunov functions are positive definite solutions to an HJ inequality. Two other uses of HJ equations include the development of numerical algorithms based on viscosity solutions (e.g., simplicial decompositions and max-plus methods) for approximating minimum cost functions, and the eikonal and shape-from-shading equations in optics and image processing. Of course it is well appreciated these days that solutions to HJ equations and even inequalities may only exist if one allows for nonsmooth solutions.

The speakers of the session are L. Grüne, A. Bacciotti, Q. Zhu, M. Malisoff, R. Goebel, and W. McEneaney. They will each present new results regarding nonsmooth solutions. Their results are summarized as follows: (1) Grüne's paper includes a new Lyapunov characterization of the input-to-state stability property; (2) Bacciotti studies the L_2 gain stability property with a discontinuous feedback law in which the feedback equation is interpreted in the sense of Filippov; (3) Zhu considers stability of general target sets, and uses lower semicontinuous Lyapunov functions to determine stable and attractive sets associated with the system; (4) Malisoff studies regularity and PDE characterizations for robust Lyapunov functions for uniformly asymptotically stable control systems using new dynamic programming methods for optimal control problems with negative instantaneous costs; (5) Goebel shows that value functions can be computed by solving a system of (generalized) Hamiltonian equations in certain situations and describes when the method will be successful; and (6) McEneaney develops new error estimates for max-plus numerical methods for estimating the value function in optimal control.

6. **Session II: Generalized Differentials and Applications**

The modern theory of HJ equations involves analyzing solutions of differential equations in some generalized manner, such as viscosity for example. When considering solutions to differential equations in a generalize sense, the usual derivative of a solution is replaced by a set-valued differential. In recent years, new theories of set-valued differentials have been developed to deal with increasingly complicated applications, such as optimization subject to differential inclusions, hybrid dynamics, or impulsive controls. This theory has been used to develop powerful new versions of the Pontryagin Maximum Principle (PMP), nonsmooth versions of Chow's Theorem, and higher-order conditions for optimality. This session will survey some of these developments and suggest avenues for further applications of set-valued differentials in applied control.

The speakers of the session are H. Sussmann, G. Colombo, P. Wolenski, B. Mordukhovich, and F. Wirth. They will each present new results on set-valued differentials and their applications in control. Summaries of their talks will be as follows: (1) Sussmann will present a forty minute tutorial on three recent generalized differentiation theories – namely, Warga's derivate containers, weak multidifferentials, and generalized differential quotients – and the applications of these theories in optimal control, such as stronger versions of the PMP. Moreover, his talk will discuss a new "path-integral generalized differential" which includes all three of these theories; (2) Colombo will discuss set-valued differentiability of "constant-dynamic" minimal time functions and its role in Lyapunov function theory and feedback stabilization; (3) Wolenski will discuss new sampling methods for impulsive systems and how nonsmooth methods can characterize set invariance properties for these systems; (4) Mordukhovich will use discrete optimization methods to obtain stability results and necessary conditions for optimality for hereditary differential inclusions; and (5) Wirth studies the (strong) time-varying stability radius of a nominal matrix A_0 and a matrix polytope P (i.e., the smallest value $r > 0$ such that the linear differential inclusion given by the set $A_0 + rP$ is not exponentially stable) using new estimates on proximal subgradients.