Alain Rapaport* (`rapaport@ensam.inra.fr`), UMR Analyse des Systèmes et Biométrie, Institut National de la Recherche Agronomique (INRA), 2 place Viala, 34060 Montpellier, France, and **Pierre Cartigny** (`cartigny@ehess.cnrs − mrs.fr`), Le Groupement de Recherche en Économie Quantitative d'Aix-Marseille (GREQAM), Université de la Méditerranée, 2 rue de la Vieille Charité, 13002 Marseille, France, *Turnpike Theorems in Infinite Horizon by a Value Function Approach*

We consider a problem of calculus of variations in infinite horizon whose objective $J(.)$ is given by

$$J(x(.)) = \lim_{T\to+\infty} \int_0^T e^{-\delta t} l(x(t), \dot{x}(t))dt$$

where δ is a positive number and l is a real valued function on $\mathbb{R} \times \mathbb{R}$, linear with respect to its second argument. Our interest is the maximization of J on the paths $x(.)$ with fixed initial condition $x(0) = x_0$, for which the preceding improper integral converges.

If the theory of calculus of variations is well established when the horizon is finite, the situation is not the same when one deals with infinite horizon. For instance, Ekeland (Lecture Notes, 1986) listed a collection of open problems in this setting, and underlined that the standard optimality conditions, such as the Euler first-order condition, have not yet been established in this framework. Our goal in this work is to study the so called Turnpike Property, which asserts, roughly speaking, that there exists a particular solution $\bar{x}(.)$ (called the "turnpike") such that, from any initial condition, an optimal trajectory reaches the path $\bar{x}(.)$ as quickly as possible.

We focus on the singular scalar case (i.e., the case where the Euler equation degenerates in an algebraic system), assuming that the integrand $l(x, y)$ is linear with respect to y and that $y \in [\alpha, \beta]$. We propose a new optimality condition of the MRAPs (Most Rapid Approach Path) which is necessary and sufficient. We consider more general growth assumptions than the usual ones. Our approach is based on a characterization of the value function of a particular Hamilton-Jacobi equation, in terms of viscosity solutions. This approach also allows us to consider the case of a multiplicity of singular solutions of the Euler equation, and therefore provides a criterion for the choice of the turnpikes in competition, depending on the initial condition. Finally we give an example which exhibits the different possible occurrences of turnpikes (one or several).