Boris Mordukhovich (boris@math.wayne.edu), Department of Mathematics, Wayne State University, Detroit, MI, *Optimal Control of Differential-Algebraic Inclusions*

This paper deals with optimal control problems for dynamical systems governed by *delayed differential-algebraic inclusions* of the type:

$$\text{minimize} \;\; J[x,y] := \varphi(x(a),x(b)) + \int_a^b f(x(t),x(t-\Delta),\dot{y}(t))dt$$

over feasible arcs $\{x(\cdot),y(\cdot)\}$ satisfying the constraints

$$\begin{array}{l} \dot{y}(t) \in F(x(t),x(t-\Delta),t) \;\; \text{a.e.} \;\; t \in [a,b], \\ y(t) = x(t) + Ax(t-\Delta), \quad t \in [a,b]. \\ x(t) = c(t), \quad t \in [a-\Delta,a), \\ (x(a),x(b) \in \Omega \subset \mathbf{R}^{2n}. \end{array}$$

Such systems are important for many engineering, economic, and ecological applications. Mathematically they are essentially different from control systems governed by ordinary differential equations and inclusions as well as by those containing time delays only in state variables. In particular, an analogue of the Pontryagin maximum principle does not hold for the differential-algebraic control systems under consideration, even in the unconstrained case with smooth dynamics and no delays, without a priori convexity assumptions on admissible velocity sets.

Our main goal is to derive necessary optimality conditions for general optimal control problems governed by differential-algebraic inclusions with endpoint constraints. To achieve this goal, we develop the method of discrete approximations that is certainly of independent interest. It allows us to build a well-posed family of discrete approximations of the original problem and establish stability results on the strong convergence of optimal solutions. Employing powerful tools of modern variational analysis and generalized differentiation, we derive necessary optimality conditions for discrete-time systems and then, by passing to the limit as the stepsize of discretization goes to zero, we arrive at new necessary optimality conditions for the original differential-algebraic systems in both Euler-Lagrange and Hamiltonian types. This talk is based on a joint work with Lianwen Wang. This research was partly supported by the National Science Foundation under grant DMS-0072179.