Daniel N. Ostrov (dostrov@scu.edu), Department of Mathematics and Computer Science, Santa Clara University, 500 El Camino Real, Santa Clara, CA 95053-0290, Nonuniqueness in Systems of Hamilton-Jacobi Equations

The scalar equation $u_t + H(f(x,t), u_x) = 0$ where H is convex and grows superlinearly in u_x has the well known property that optimal control paths for its solution (i.e., characteristic paths) cannot enter a shock as the path progresses towards the equation's initial condition if f is a continuous function. On the other hand these paths can enter shocks if f is a discontinuous function. This phenomenon will allow us to see how the solution for some viscous non-strictly hyperbolic decoupled systems of the form

$$u_t + H_1(u_x) = \varepsilon_1 u_{xx}$$
$$v_t + H_2(u_x, v_x) = \varepsilon_2 v_{xx},$$

is not unique as $(\varepsilon_1, \varepsilon_2) \to (0, 0)$; that is, the solution depends on how $(\varepsilon_1, \varepsilon_2)$ approach (0, 0). If time permits we will also examine some of the difficulties in trying to use control theory to describe unique solutions to strictly hyperbolic systems of Hamilton-Jacobi equations.