Franco Rampazzo* and **Monica Motta** ({`rampazzo`, `motta`}`@math.unipd.it`), Dipartimento di Matematica Pura ed Applicata, Università di Padova, ITALY, *Multitime Hamilton-Jacobi systems*

We investigate existence and uniqueness of the (viscosity) solution for systems of convex Hamilton-Jacobi equations of the form

$$\begin{cases} u_{t_1}(t_1,...,t_d,x) + H_1(x, Du(t_1,...,t_d,x)) = 0 \\ .... \\ .... \\ u_{t_d}(t_1,...,t_d,x) + H_d(x, Du(t_1,...,t_d,x)) = 0 \end{cases} \qquad (t_1,...,t_d,x) \in]0,T[^d \times \mathbf{R}^n \tag{1}$$

satisfying the initial condition

$$u(0,...,0,x) = \psi(x)$$

These systems are called *multi-time* in an earlier paper by P.Lions and J.C.Rochet. An application in economics of these system is presented in [R].

In general system (1) is overdetermined. Actually this is not the case when the Hamiltonians H_i do not depend on the state variable x. In fact, this case is investigated in [LR], where the authors utilize a commutativity property for semigroups to prove existence of a *weak* solution to (1). Their tools are essentially Oleinik-Lax formulas. A generalization of this result to the case when the Hamiltonians depend un u as well (but not on x) has been recently provided by S. Plaskacz and M. Quincampoix in [PQ]. They exploit Oleinik-Lax formulas up to the point they can allow semicontinuous solutions.

Of course, the problem with x-dependent Hamiltonians has a richer underlying geometry, and, to our knowledge, it has been investigated only in a recent paper by G. Barles and A. Tourin [BT]. They prove existence of a viscosity solution under the assumptions that the Hamiltonians are C^1 and convex in the gradient variable, and, moreover, the involution property

$$\{H_i, H_j\} = (H_i)_x (H_j)_p - (H_j)_x (H_i)_p = 0 \qquad \forall i,j = 1,...d \tag{2}$$

is verified on the whole domain.

Of course hypothesis (2) is satisfied in the x-independent case. Furthermore it can be shown that (2) is necessary for the existence of even a local solution of (1). However, as the same authors point out, with the methods exploited in [BT] the C^1-regularity assumption is hardly removable.

Here we are able to handle a class of *Lipschitz continuous* Hamiltonians arising in Control Theory (moreover, we can drop a coercivity- type condition assumed in [BT] mostly for technical reasons). Our approach is based on a combination of control-theoretical techniques and arguments from the theory of viscosity solutions. Moreover, in order to allow nonsmoothness, we utilize a recent result by F. Rampazzo and H. Sussmann on the commutativity of Lipschitz continuous vector fields.

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