Gaemus Collins (gcollins@ucsd.edu), Department of Mathematics, University of California, San Diego, CA 92093-0112, *Min-Plus Eigenvector Methods for Nonlinear H_∞ Problems with Active Control*

In [1] McEneaney considers the H_∞ problem for a nonlinear system. The dynamic programming equation (DPE) is a fully nonlinear, first order, steady state partial differential equation (PDE), posessing a term which is quadratic in the gradient. The solutions are typically nonsmooth, and further, there is non-uniqueness among the class of viscosity solutions. In the case where one tests a fixed-feedback control to see if it yields an H_∞ controller, the PDE is a Hamilton-Jacobi-Bellman equation. In the case where the "optimal" feedback control is being determined as well, the problem takes the form of a differential game, and the PDE is, in general, an Isaacs equation. The computation of the solution of a nonlinear, steady-state, first-order PDE is typically quite difficult. In [1] McEneaney began the development of an entirely new class of methods for obtaining the "correct" solution of such PDEs. The focus was on the fixed-feedback case. The methods were based on the linearity of the associated semigroup over the max-plus algebra. In particular, the solution of the PDE was reduced to the solution of a max-plus eigenvector problem for the known unique eigenvalue 0 (the max-plus multiplicative identity). It was demonstrated that the eigenvector is unique and that the power method converges to it.

McEneaney briefly considers the problem with active control, that is, where the feedback is unknown. It is conjectured that similar results to the fixed-feedback control case should hold. In this paper the corresponding value function for the active control problem is shown to satisfy a similar PDE as in the fixed-feedback control case. We assume that the controller can dominate the disturbance input. We prove the semigroup associated with this new PDE is linear over the min-plus algebra, the solution is semi-concave, and the associated min-plus eigenvalue problem not only gives the solution to the PDE, but can also be solved by the power method.

[1] W. McEneaney, "Max-plus eigenvector representations for solution of nonlinear H_∞ problems: basic concepts," *IEEE Transactions on Automatic Control*, submitted.