Héctor J. Sussmann (sussmann@hilbert.rutgers.edu), Department of Mathematics, Rutgers University, *Finite-Dimensional Generalized Differentiation Theories that Satisfy the Chain Rule and Have an Open Mapping Property*

The classical differential of maps of class C^1 is a functor from the category PMC^1 of pointed manifolds of class C^1 (i.e., pairs (M,p) consisting of a manifold M of class C^1 and a distinguished point p of M) and distinguished-point-preserving maps of class C^1 to the category $FDRLS$ of finite-dimensional real linear spaces and linear maps. Furthermore, this functor has an open mapping property: if the differential of a map F at a point p is surjective, then the map itself sends neighborhoods of p to neighborhoods of $F(p)$. We study extensions of this functor to categories with the same objects as PMC^1 but larger classes of morphisms (including Lipschitz maps, many continuous maps that are not Lipschitz, and many set-valued maps), taking values in a larger category (whose morphisms are nonempty compact sets of linear maps), and still having the open mapping property in a suitable sense. These extensions are really "multivalued functors" (as might perhaps have been expected for a theory that belongs to the general field of set-valued analysis), and the functoriality property is the Chain Rule. A number of such extensions had been constructed in previous work by several authors, including the speaker, but they were not mutually comparable. We propose an extension, called "path-integral generalized differentials", that contains all the others and still obeys the chain rule and has an open mapping property. We also discuss the harder problem of working with a category with a larger class of objects, e.g. pairs (C,p) where C is a subset of a manifold M of class C^1 which is, near p, C^1-diffeomorphic to a closed convex cone, and show how the theory of path-integral generalized differentials also work in this case, at least if one limits oneself to sets C that are C^1-diffeomorphic to polyhedral cones.