

**Matthias Kawski** ([kawski@asu.edu](mailto:kawski@asu.edu)), Department of Mathematics and Statistics, Arizona State University, Tempe, Arizona 85287, *Nonconvex Needle Variations*

We discuss a recently found counter-example (joint work with R. M. Bianchini) which shows that, in general, sets of tangent vectors generated by needle variations may fail to lack desirable convexity properties, even for the most benign systems. Our system is a four-input polynomial cascade system in a six-dimensional state space. Explicit constructions show that certain two-dimensional cross-sections of the sets of tangent vectors generated by needle variations are unions of two non-identical half-spaces [*sic*].

The focus of this presentation is on the mechanism that practically *fixes* the needle variations at a point: Recall that smooth dependence on initial conditions allows that a family of needle variations can be moved by a distance that decreases to zero, and still generate the same tangent vector. We show that the key mechanism in our example is that either of two families of needle variations – which cannot be combined – can only be moved by an amount that decreases at a rate faster than linearly to zero when compared to length of the needle variation.