Hedy Attouch and **Marc-Olivier Czarnecki*** (`attouch@math.univ-montp2.fr` and `marco@math.univ-montp2.fr`), Laboratoire d'Analyse Convexe, case courier 51, Universite de Montpellier 2, Place Eugene Bataillon, 34095 Montpellier Cedex 5, France, *Asymptotic Control and Stabilization of Nonlinear Oscillators with Non-Isolated Equilibria*

Let $\Phi : H \to \mathbf{R}$ be a $\mathcal{C}^1$ function on a real Hilbert space H and let $\gamma > 0$ be a positive (damping) parameter. For any control function $\varepsilon : \mathbf{R}_+ \to \mathbf{R}_+$ which tends to zero as $t \to +\infty$, we study the asymptotic behavior of the trajectories of the damped nonlinear oscillator

$$(HBFC) \quad \ddot{x}(t) + \gamma \dot{x}(t) + \nabla\Phi(x(t)) + \varepsilon(t)x(t) = 0.$$

We show that, if $\varepsilon(t)$ does not tend to zero too rapidly as $t \to +\infty$, then the term $\varepsilon(t)x(t)$ asymptotically acts as a Tikhonov regularization, which forces the trajectories to converge to a particular equilibrium. Indeed, in the main result of this paper, it is established that, when Φ is convex and $S = \operatorname{argmin} \Phi \neq \emptyset$, under the key assumption that ε is a "slow" control, i.e., $\int_0^{+\infty} \varepsilon(t)dt = +\infty$, then each trajectory of the $(HBFC)$ system strongly converges, as $t \to +\infty$, to the element of minimal norm of the closed convex set S. As an application, we consider the damped wave equation with Neumann boundary condition

$$\begin{cases} u_{tt} + \gamma u_t - \Delta u + \varepsilon(t)u(t) = 0 & \text{in} \quad \Omega \times \mathbf{R}_+, \\ \dfrac{\partial u}{\partial \mathbf{n}} = 0 & \text{on} \quad \partial\Omega \times \mathbf{R}_+. \end{cases}$$