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We present applications of parametric variational principles (Ekeland’s and Borwein-Preiss’ variational principles) to optimal control problems. We show that, in some classical optimal control problems, there exists a solution depending continuously on a parameter. Some of the results are based on the following lemma, which has independent interest.

Lemma (Parametric ε -variational principle) Suppose that X is a paracompact topological space, E is a Banach space, $Y \subset E$ is a closed, convex and nonempty subset, $F : X \rightarrow 2^Y$ is lower semi-continuous multivalued mapping with convex nonempty images, $\varepsilon > 0$ is given and the functions $f : X \times Y \rightarrow \mathbf{R}$, $g : X \rightarrow \mathbf{R}$ satisfy the conditions:

- (1) $f(x, \cdot)$ is quasi-convex for every $x \in X$;
- (2) $f(\cdot, y)$ is upper semi-continuous for every $y \in Y$;
- (3) g is lower semi-continuous and $g(x) \geq \inf_{y \in (F(x) + \varepsilon B) \cap Y} f(x, y)$ for every $x \in X$ (B is the closed unit ball).

Then there exists a continuous selection $\varphi_\varepsilon : X \rightarrow Y$ of the mapping $F_\varepsilon(x) = F(x) + \varepsilon B$ such that $f(x, \varphi_\varepsilon(x)) < g(x) + \varepsilon \quad \forall x \in X$.