Tzanko Donchev (tdd51us@yahoo.com), Department of Mathematics, University of Architecture & Civil Engineering, 1046 Sofia, Bulgaria, *Relaxed One Sided Lipschitz Multifunctions and Applications*

We present an overview of the so-called Relaxed One Sided Lipschitz (ROSL) multifunctions and their applications to differential inclusions

$$\dot{x}(t) \in F(t, x(t)),\ x(0) = x_0,\ t \in I := [0, 1]. \tag{1}$$

Given a Hilbert space E, a bounded valued multifunction F from $I \times E$ into E is said to be **Relaxed One Sided Lipschitz (ROSL)** with a constant L provided

$$\sigma(x - y, F(t, x)) - \sigma(x - y, F(t, y)) \leq\ L|x - y|^2 \tag{2}$$

for a.e. $t \in I$ and each $x, y \in E$, where $\sigma(\cdot, B)$ is the support function of the set B. If E is a Banach space with uniformly convex dual E^*, then the inequality (2) becomes

$$\sigma(J(x - y), F(t, x)) - \sigma(J(x - y), F(t, y)) \leq\ L|x - y|^2,$$

where $J(\cdot)$ is the normalized duality map. When E is an arbitrary Banach space, however, one must use the following definition:

For every $x, y \in E$ and a.e. $t \in I$:

If $f_x \in F(t, x)$, then for every $\varepsilon > 0$ there exists $f_y \in F(t, y)$ such that

$$[x - y, f_x - f_y]_+ < L|x - y| + \varepsilon,$$

where $[x, y]_+ = \lim\limits_{h \to 0^+} h^{-1}\{|x + hy| - |x|\}$.

It is easy to see that the ROSL condition is essentially weaker than Lipschitz continuity and relaxes essentially the “classical” OSL condition:

$$[x - y, f_x - f_y]_+ \leq L|x - y|,\ \forall f_x \in F(t, x),\ f_y \in F(t, y).$$

Moreover, if F is Lipschitz, then it is possible that the ROSL constant can be less than the Lipschitz one. The following theorem is a standard result for the differential inclusion (1):

Theorem 0.1 *Let $\overline{\text{co}}\ F(\cdot, \cdot)$ be almost continuous, with nonempty compact values, and bounded on bounded sets. If $F(t, \cdot)$ is ROSL with closed values, then the solution set of*

$$\dot{x}(t) \in \overline{\text{co}}\ F(t, x(t)),\ x(0) = x_0 \tag{3}$$

is a nonempty R_δ set. Furthermore, the solution set of (1) is nonempty, connected and dense in the solution set of (3).

We consider several other applications of the ROSL condition and present illustrative examples.