The Newton Bracketing Method for Minimizing Convex Functions

Let $f$ be a convex function of $n$ variables, bounded below, with infimum attained. A bracket is an interval $[L, U]$ containing the attained infimum $\min f$. The Newton Bracketing (NB) method \cite{1} is an iterative method that at each iteration does a single Newton iteration and then reduces the bracket by either raising $L$ or lowering $U$. An initial lower bound $L$ must be provided, and the initial upper bound is $U(x)$ where $x$ is the initial iterate.

Unlike gradient methods (and other iterative methods that directly drive $x$ to optimum), the NB method has a natural stopping criterion: the bracket size. Numerical experience reported here shows the average bracket reduction per iteration is about $1/2$, so convergence is fast.

The method is valid for $n = 1$. It is valid for $n > 1$ (using the directional Newton iteration \cite{2}) provided the level sets of $f$ are not "too narrow". A precise statement for quadratic $f$, i.e., $f = \frac{1}{2}x^TQx + \text{linear terms}$, with $Q$ positive definite, is: the NB method is valid if the condition number of $Q$ is $< 1/(7 - \sqrt{48}) \approx 14$.

The method was applied in \cite{1} for solving location (Fermat-Weber) problems, and is applied here to location problems with affine constraints, and to affinely constrained convex programs in general. Applications to trust region subproblems in SDP are developed by H. Wolkowicz.
