Stabilization of a Periodic Trajectory for a Chemostat with Two Species

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Joint with Frédéric Mazenc (Projet MERE INRIA-INRA)
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- Background and Objectives
- Local Stabilization
- Numerical Validation
- Global Stabilization
- Strict Lyapunov Function
- Conclusions and Further Research

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BACKGROUND and GOAL

Basic Model: The two-species chemostat with nutrient concentration $s(t)$ and organism concentrations $x_i(t)$ evolving on $[0, \infty) \times (0, \infty)^2$ is

$$\begin{cases} \dot{s} & = & D[s_{in} - s] - \sum_{j=1}^{2} \mu_j(s) x_j \ , \\ \dot{x}_i & = & [\mu_i(s) - D] x_i \ , \quad i = 1, 2 \end{cases} \qquad (\Sigma_c)$$

where $D(\cdot)$ is the dilution rate, $s_{in}(\cdot)$ is the concentration of the input nutrient, and $\mu_1, \mu_2 : [0, \infty) \to [0, \infty)$ are uptake functions.

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Competitive Exclusion: When $s_{in}(\cdot)$ and D are constant and the μ_i's are increasing, at most one species survives. (There is a steady state with at most one nonzero species concentration, which attracts a.a. solutions.)

OVERVIEW of LITERATURE

Coexistence: In real ecological systems, $n > 1$ species can coexist, so much of the literature aims at choosing s_{in} and/or D to force coexistence.

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Input-to-State Stability: Mazenc-M-De Leenheer (*CDC*'06, *MBE*'07) designed feedbacks D and s_{in} and Lyapunov functions for one species chemostats that gave (i)ISS tracking relative to actuator errors.

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LOCAL STABILIZATION RESULT

Assumption A1: $\mu_i(0) = 0$, $\mu_i \in C^1$, $\mu_i' > 0$ bounded for $i = 1, 2$. $\exists s_c > 0$ such that $\chi(s) := \mu_2(s) - \mu_1(s)$ satisfies $\chi(s_c) = 0$, $\chi(s) < 0$ when $0 < s < s_c$, $\chi(s) > 0$ when $s > s_c$, and $\chi'(s_c) > 0$. $\Gamma = \mu_1(s_c)$.

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Reference Trajectory: Given any constant $\alpha \in [0, \Gamma)$, we wish to track $(s_r, x_{1r}, x_{2r}) = (s_c, \exp(\cos(\alpha t)), \exp(\cos(\alpha t)))$, i.e., stabilize the error $(s_e, \xi_{1e}, \psi) := (s - s_c, \xi_1 - \cos(\alpha(t)), \xi_2 - \xi_1)$ to 0, where $\xi_i = \ln(x_i)$. More general reference trajectories are tractable by analogous methods.

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Theorem 1: Under Assumption A1, the control laws

$$D(t, \xi_1) \ := \ \Gamma + \alpha \sin(\alpha t) + \frac{(\Gamma-\alpha)^2}{\Gamma}(\xi_1 - \cos(\alpha t))$$

$$s_{in}(t) := s_c + \frac{2\Gamma e^{\cos(\alpha t)}}{\Gamma + \alpha \sin(\alpha t)}$$

render the error dynamics locally exponentially stable to 0.

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render the error dynamics locally exponentially stable to 0. Hence, they locally exponentially stabilize the reference trajectory.

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EXAMPLE

We track $(s_r, x_{1r}, x_{2r}) = (0.9, \exp(\cos(t/4)), \exp(\cos(t/4)))$ for

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Our assumptions are satisfied using

$$\mu_1(s) = \frac{10s}{1+20s}, \quad \mu_2(s) = \frac{s}{1+s}, \quad s_c = \frac{9}{10}, \quad \text{and} \quad \Gamma = \frac{9}{19}.$$

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Therefore, we get the locally uniformly stabilizing controllers

$$\begin{array}{rcl} D(t, \xi_1) & = & \frac{9}{19} + \frac{1}{4}\sin(t/4) + \frac{19}{9}\left(\frac{9}{19} - \frac{1}{4}\right)^2 (\xi_1 - \cos(t/4)) \\ s_{in}(t) & = & \frac{9}{10} + \frac{72e^{\cos(t/4)}}{36+19\sin(t/4)} \end{array}$$

which cause the trajectories of (Σ_e) to locally track the green trajectory.

SIMULATION for (Σ_e) with $(s, x_1, x_2)(0) = (.001, 10, .001)$

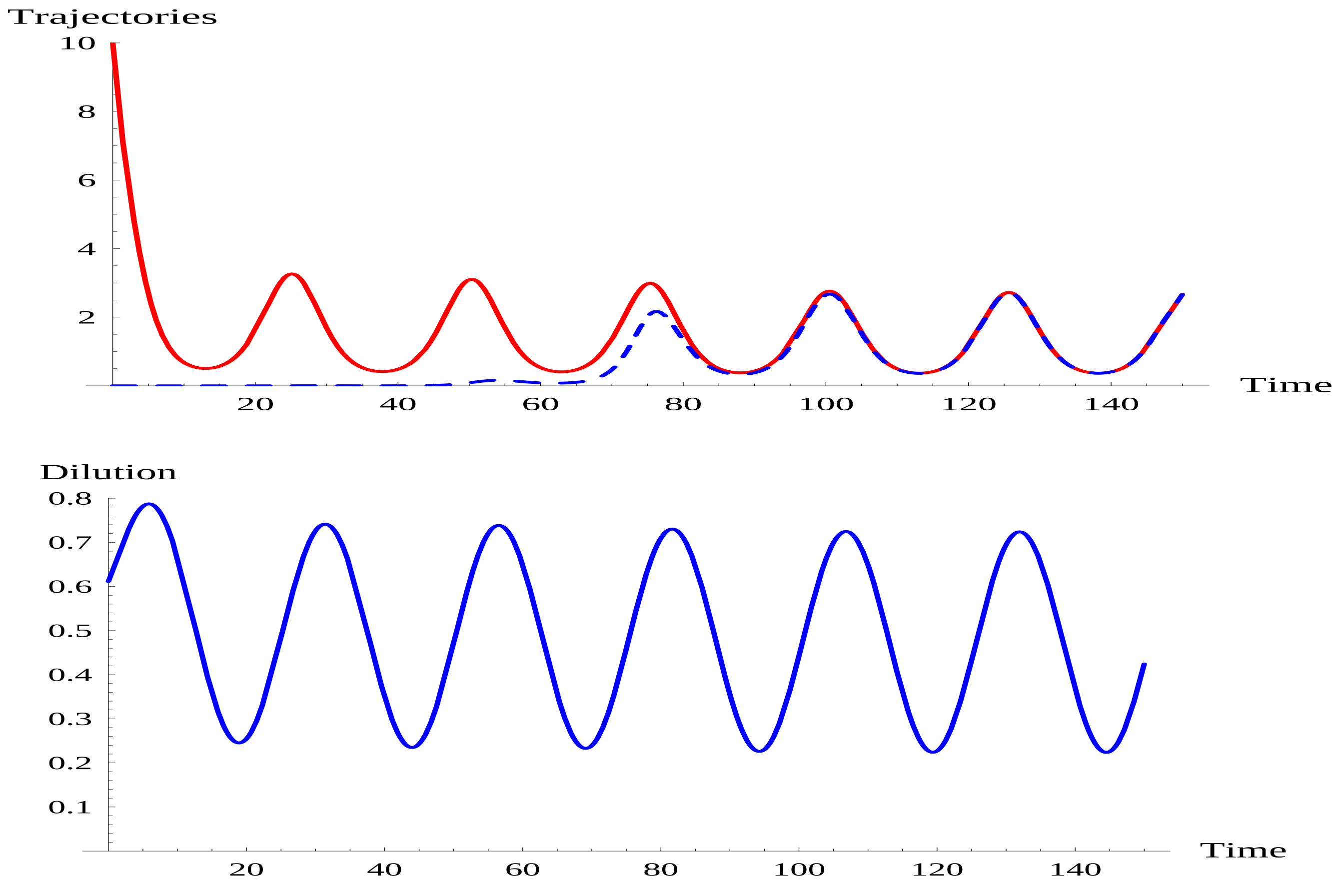

Figure 1: x_1 (solid red line). x_2 (dashed blue line). D (solid blue line).

SIMULATION for (Σ_e) with $(s, x_1, x_2)(0) = (10, .015, 10)$

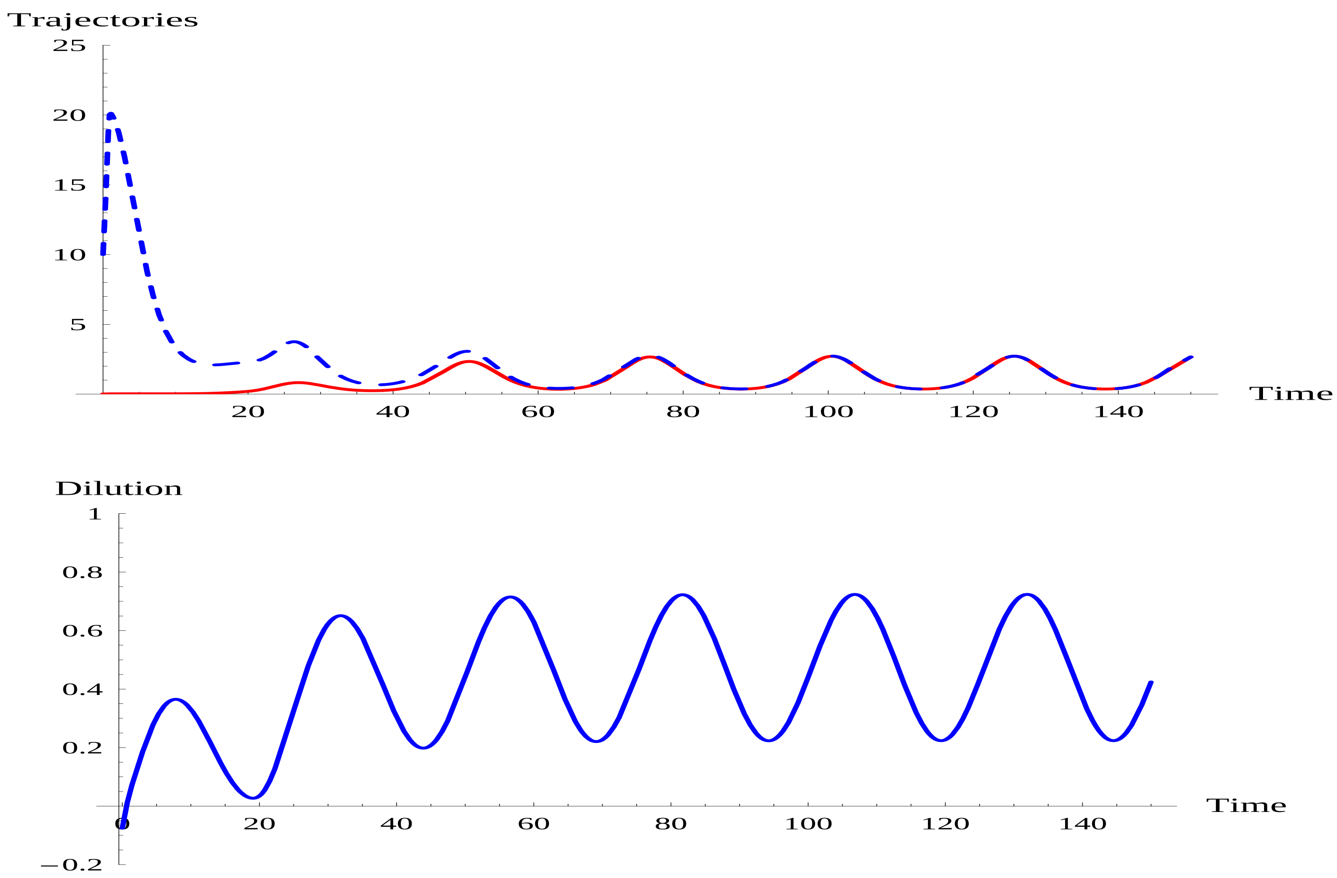

Figure 2: x_1 (solid red line). x_2 (dashed blue line). D (solid blue line).

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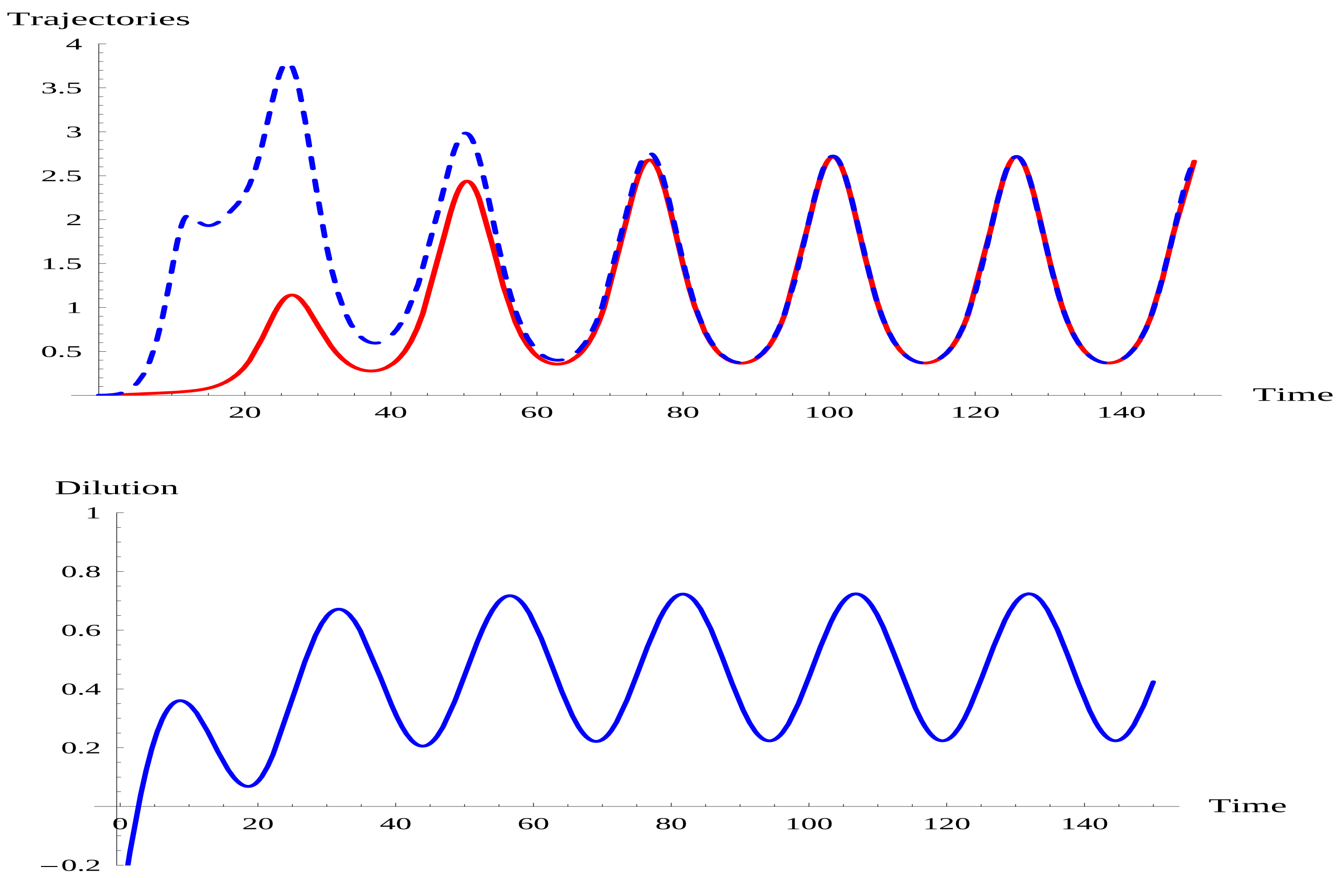

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Assumption A2: $\mu_1, \mu_2 \in C^2$. $\exists \theta_1, \theta_2 > 0$ such that

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Under Assumptions A1-A2, we define the constants

$$c_1 := \tfrac{\Gamma}{16} \min\left\{ \tfrac{s_c}{\mu_1'(s_c)}, \tfrac{1}{\theta_1} \right\}, \quad c_2 := \tfrac{\Gamma}{16} \min\left\{ \tfrac{s_c}{\mu_2'(s_c) - \mu_1'(s_c)}, \tfrac{1}{\theta_1 + \theta_2} \right\}$$

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render the error dynamics GAS and locally exponentially stable to the origin.

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STRICT LYAPUNOV FUNCTION

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has the nonstrict Lyapunov function

$$V(s_e, \xi_{1e}, \psi) := \frac{1}{2}s_e^2 + c_1\left(\sqrt{1 + \xi_{1e}^2} - 1\right) + c_2\left(\sqrt{1 + \psi^2} - 1\right)$$

i.e. $\dot{V} \leq -\frac{\Gamma}{8}s_e^2 - \frac{\Gamma}{4}c_1\langle\xi_{1e}\rangle^2 \leq 0$ along the error dynamics.

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Step 2: Construct a positive increasing function κ so that

$$V_a(s_e, \xi_{1e}, \psi) \ := \ s_e\langle\psi\rangle + \int_0^{V(s_e,\xi_{1e},\psi)} \kappa(r)dr$$

is a strict Lyapunov function for the error dynamics.

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