1. Evaluate $\int\int_D x \, dA$ using polar coordinates. The region $D$ is the wedge defined by $0 \leq r \leq 2$, $0 \leq \theta \leq \frac{\pi}{6}$. 

\[ x = r \cos \theta \] and $dA = r \, dr \, d\theta$. So 

\[ \int\int_D x \, dA = \int_0^{\pi/6} \int_0^2 r \cos \theta \, r \, dr \, d\theta \]

\[ = \int_0^{\pi/6} \int_0^2 \cos \theta \, r^2 \, dr \, d\theta \]

\[ = \int_0^{\pi/6} \cos \theta \frac{r^3}{3} \bigg|_0^2 \, d\theta \]

\[ = \frac{8}{3} \sin \theta \bigg|_{\theta=0}^{\theta=\pi/6} \]

\[ = \frac{8}{3} \sin \frac{\pi}{6} \]

\[ = \frac{4}{3}. \]

2. Find the area on the surface $z = \frac{2}{3}x^{3/2} + y + 1$ above the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 1$ in the $x$-$y$ plane.

\[ dS = \sqrt{z_x^2 + z_y^2 + 1} \, dx \, dy. \]

Now $z_x = x^{1/2}$ and $z_y = 1$. So $dS = \sqrt{x + 2} \, dx \, dy$. Thus 

\[ S = \int_0^1 \int_0^2 (x + 2)^{1/2} \, dx \, dy \]

\[ = \int_0^1 \frac{2}{3}(x + 2)^{3/2} \bigg|_{x=0}^{x=2} \, dy \]

\[ = \int_0^1 \frac{2}{3}(4)^{3/2} - \frac{2}{3}(2)^{3/2} \, dy \]

\[ = \frac{2}{3} (8 - 2\sqrt{2}) \]

3. Evaluate $\int\int\int_Q x \, dV$

where $Q$ is the solid bounded by the surfaces $z = 1 - x^2$, $z = 0$, $y = 0$, $y = 1$, $x = 0$, $x = 1$. 

\[ \int_0^1 \int_0^1 \int_0^{1-x^2} x \, dz \, dy \, dx = \int_0^1 \int_0^1 x(1 - x^2) \, dy \, dx = \int_0^1 x(1 - x^2) \, dx = -\frac{1}{4}(1 - x^2)^2 \bigg|_0^1 = \frac{1}{4}. \]
4. Find the volume using cylindrical coordinates (or polar coordinates) of the solid inside the cylinder \( x^2 + y^2 = 4 \) and bounded on top and bottom by the surface \( z^2 - x^2 - y^2 = 1 \).

Note the region in the \( x-y \) plane that this solid lives above and below is the disk \( 0 \leq r \leq 2 \), \( 0 \leq \theta \leq 2\pi \). \( z^2 - x^2 - y^2 = 1 \) is a two sheeted hyperboloid. The upper sheet has equation \( z = \sqrt{1 + x^2 + y^2} \) and the lower sheet has equation \( z = -\sqrt{1 + x^2 + y^2} \). In cylindrical coordinate we have \( z = \sqrt{1 + r^2} \) for the upper sheet and \( z = -\sqrt{1 + r^2} \) for the lower sheet. Hence since \( dV = r \, dz \, d\theta \, dr \),

\[
V = \iiint_Q dV = \int_0^2 \int_0^{2\pi} \int_{-\sqrt{1+r^2}}^{\sqrt{1+r^2}} r \, dz \, d\theta \, dr \\
= \int_0^2 \int_0^{2\pi} 2r \sqrt{1 + r^2} \, d\theta \, dr \\
= \int_0^2 (1 + r^2)^{\frac{3}{2}} (2r) \, dr \\
= 2\pi \int_0^2 (1 + r^2)^{\frac{3}{2}} \, dr \\
= 2\pi \cdot \frac{2}{3} (1 + r^2)^{\frac{3}{2}} \bigg|_0^2 \\
= \frac{4\pi}{3} (5^{\frac{3}{2}} - 1)
\]

5. Give an iterated integral for the volume inside the sphere \( x^2 + y^2 + z^2 = 25 \) outside the cylinder \( x^2 + y^2 = 9 \). You may wish to use cylindrical coordinates. Do not evaluate.

We use cylindrical coordinates. Note since we are outside the cylinder \( x^2 + y^2 = 9 \) and inside the sphere \( x^2 + y^2 + z^2 = 25 \), we have \( 3 \leq r \leq 5 \). Also \( 0 \leq \theta \leq 2\pi \). Finally for each \( r \) and \( \theta \), \( z \) varies from \( z = -\sqrt{25 - x^2 - y^2} = -\sqrt{25 - r^2} \) to \( z = \sqrt{25 - x^2 - y^2} = \sqrt{25 - r^2} \). Since \( dV = r \, dz \, d\theta \, dr \), we have

\[
V = \iiint_Q dV = \int_3^5 \int_0^{2\pi} \int_{-\sqrt{25-r^2}}^{\sqrt{25-r^2}} r \, dz \, d\theta \, dr.
\]