Uniform Global Asymptotic Stability of Adaptive Cascaded Nonlinear Systems with Unknown High-Frequency Gains

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Joint with Frédéric Mazenc and Marcio de Queiroz
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Both are shown by constructing specific kinds of strict Lyapunov functions for $\dot{Y} = \mathcal{G}(t, Y, 0)$.

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$$\mathcal{P}_i \stackrel{\text{def}}{=} \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \in \mathbb{R}^{(p_i+1)\times(p_i+1)}, \tag{9}$$

  where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for each i.

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$$u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi}) = \frac{v_{f,i}(t,\tilde{\xi})-g_i(\xi)-k_i(\xi)\cdot\hat{\theta}_i+\dot{z}_{R,i}(t)}{\hat{\psi}_i} \tag{14}$$

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$$\begin{array}{rcl}\varpi_{i,j} & = & -\frac{\partial V}{\partial\tilde{z}_i}(t,\tilde{\xi})k_{i,j}\big(\tilde{\xi}+\xi_R(t)\big)\ \ \text{and}\\ \mho_i & = & -\frac{\partial V}{\partial\tilde{z}_i}(t,\tilde{\xi})u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi})\ .\end{array} \tag{13}$$

$$u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi})\ =\ \frac{v_{f,i}(t,\tilde{\xi})-g_i(\xi)-k_i(\xi)\cdot\hat{\theta}_i+\dot{z}_{R,i}(t)}{\hat{\psi}_i} \tag{14}$$

The estimator and feedback can only depend on things we know.

Stabilization Analysis

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- We build a global strict Lyapunov function for the $Y = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ dynamics to prove the UGAS condition $|Y(t)| \leq \gamma_1(e^{t_0 - t}\gamma_2(|Y(t_0)|))$.

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- We start with the nonstrict Lyapunov function

$$V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\widetilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \mathrm{d}m + \sum_{i=1}^{s} \int_0^{\widetilde{\psi}_i} \frac{m}{(\psi_i - m - \underline{\psi})(\overline{\psi} - \psi_i + m)} \mathrm{d}m \,.$$

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Theorem: We can construct $K \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \doteq K\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Upsilon}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \ , \tag{15}$$

$$\begin{aligned} \text{where} \quad \overline{\Upsilon}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) &= -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \\ &\quad + \tfrac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \ , \end{aligned} \tag{16}$$

$$\lambda_i(t) = \left(k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t))\right) \ , \tag{17}$$

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- The unknown vectors $\beta_1 \in \mathbb{R}^2$ and $\beta_2 \in \mathbb{R}^3$ and unknown scalars γ_1 and γ_2 are the motor electric parameters.

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- It would be useful to extend to cover models that are not affine in θ, feedback delays, and output feedbacks.