Adaptive Tracking and Parameter Identification: Theory and Marine Robotic Applications

Michael Malisoff

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ZP. Jiang, E. Justh, P. Krishnaprasad, V. Lumelsky, A. Stepanov

Marine Robots (with Georgia Tech)

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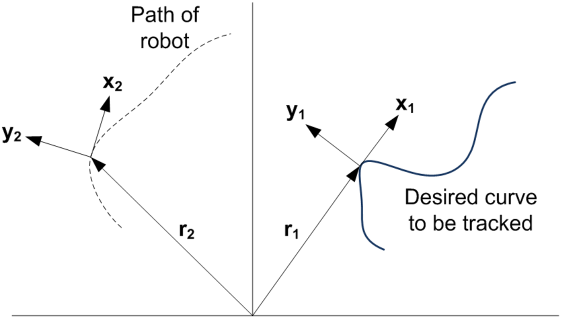
Simpler 2D case: Boundary following with gyroscopic control.



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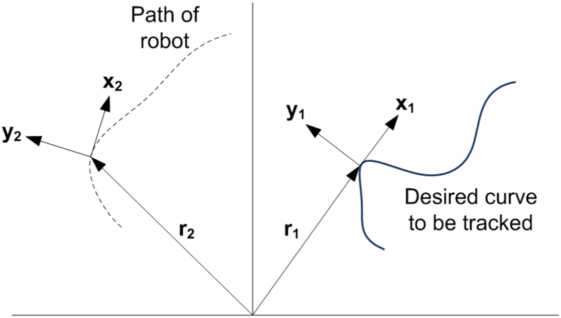

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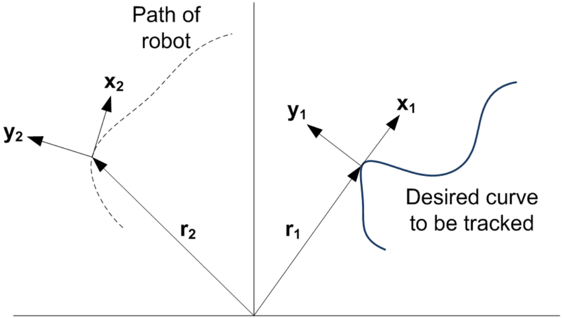

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$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

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$$U(\rho,\phi) = -h'(\rho)\sin(\phi) + \frac{1}{\mu}\int_0^{V(\rho,\phi)} \Gamma_0(m)\mathrm{d}m \tag{7}$$

Strictification (Mazenc-M-Zhang, TAC)

Theorem 1: The closed loop system (CL) has the strict Lyapunov function

$$U(\rho,\phi) =$$
$$-h'(\rho)\sin(\phi) + \frac{1}{\mu}\int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m + \Gamma(V(\rho,\phi)) + V(\rho,\phi),$$

where $\gamma(q) = \frac{2(q+2\rho_0)^3}{\rho_0^4} + 1 + 0.5\mu^2 + \mu,$

$\Gamma(q) = \frac{18}{\rho_0}q + 9\left(\frac{2}{\rho_0}\right)^4 q^4,$ and $V(\rho,\phi) = -\ln\left(\cos(\phi)\right) + h(\rho)$

on its state space $\mathcal{X} = (0,+\infty) \times (-\pi/2, \pi/2)$. $\square$

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$$U(\rho(t),\phi(t)) \geq V(\rho(t),\phi(t)) \qquad \text{(PD)}$$

$$\frac{d}{dt}U(\rho(t),\phi(t)) \leq -0.5[h'(\rho(t))\cos(\phi(t))]^2 - \sin^2(\phi(t)) \qquad \text{(SD)}$$

Unknown Control Gains (M-Zhang)

$$\begin{cases} \dot{\rho} & = & -\sin(\phi) \\ \dot{\phi} & = & \frac{\kappa \cos(\phi)}{1+\kappa\rho} + K u \end{cases} \tag{8}$$

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for the augmented error $(\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K)$ on each set $\mathcal{S}_i \times (c_{\min} - K, c_{\max} - K)$ where $\mathcal{S}_i$ is a nested sequence of compact sets that fill our state space $\mathcal{X}$ and $c_{\min} < K < c_{\max}$.

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Strict Decay: $\dot{V}^\sharp \leq -\alpha_0(V^\sharp),$ with α_0 positive definite (13)

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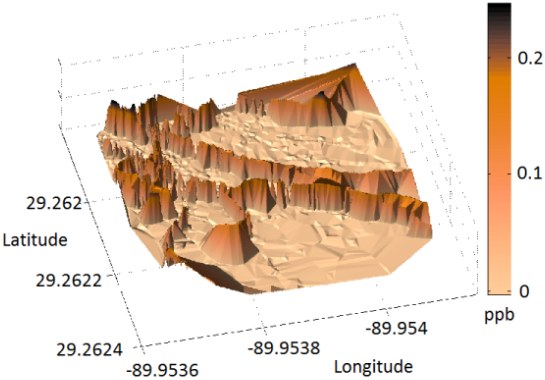

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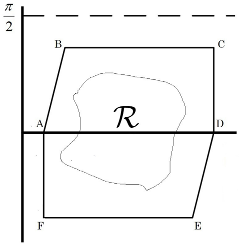
$\bar{c}, \underline{c}$: Known upper and lower bounds on unknown κ_0

Nested Hexagons that Fill $\mathcal{X}$



AB and DE have slope μ

Others flat or vertical

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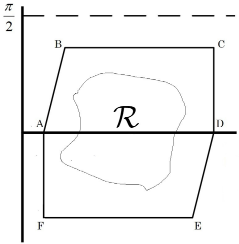

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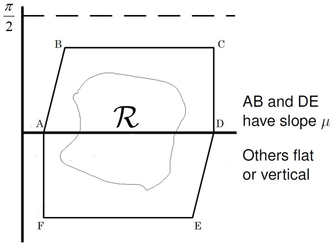

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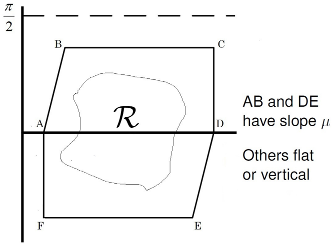
$D = (\rho_* + \bar{K}\rho_0, 0)^\top$, $E = (\bar{K}\rho_0, -\mu\rho_*)^\top$, and $F = (\rho_*, -\mu\rho_*)^\top$

Fix one such hexagon $\mathcal{S}$ and a perturbation bound $\bar{\Delta}$ such that

$$\dot{\rho} \; = \; -\sin(\phi), \quad \dot{\phi} \; = \; h'(\rho)\cos(\phi) - \mu\sin(\phi) + \delta(t) \tag{17}$$

has $\mathcal{S}$ as a forward invariant set for all $\delta : [0, \infty) \to [-\bar{\Delta}, \bar{\Delta}]$.

Nested Hexagons that Fill $\mathcal{X}$



$A = (\rho_*, 0)^\top$, $B = (2\rho_*, \mu\rho_*)^\top$, $C = (\rho_* + \bar{K}\rho_0, \mu\rho_*)^\top$,
$D = (\rho_* + \bar{K}\rho_0, 0)^\top$, $E = (\bar{K}\rho_0, -\mu\rho_*)^\top$, and $F = (\rho_*, -\mu\rho_*)^\top$

Fix one such hexagon $\mathcal{S}$ and a perturbation bound $\bar{\Delta}$ such that

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has $\mathcal{S}$ as a forward invariant set for all $\delta : [0, \infty) \to [-\bar{\Delta}, \bar{\Delta}]$.

Augmented Error: $(\tilde{q}, \tilde{\kappa}_0) = (\tilde{q}_1, \tilde{q}_2, \tilde{\kappa}_0) = (\rho - \rho_0, \phi, \hat{\kappa}_0 - \kappa_0)$

Curvature Identification (M-Sizemore-Zhang)

$$\begin{cases} \dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 &= h'(\tilde{q}_1+\rho_0)\cos(\tilde{q}_2)-\mu\sin(\tilde{q}_2) \\ & \quad -\frac{\tilde{\kappa}_0\cos(\phi)}{(1+\kappa_0(\rho-\rho_0))(1+(\kappa_0+\tilde{\kappa}_0)(\rho-\rho_0))} \\ \dot{\tilde{\kappa}}_0 &= \frac{(\kappa_0+\tilde{\kappa}_0-\underline{c})(\bar{c}-\kappa_0-\tilde{\kappa}_0)}{(1+(\rho-\rho_0)\hat{\kappa}_0)^2}\cos(\tilde{q}_2)\frac{\partial U}{\partial\phi}(\rho,\phi). \end{cases} \tag{18}$$

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$$\begin{aligned} &\bar{\mathcal{M}}_1 \geq \tfrac{1}{\mu}\gamma(V(\rho,\phi)) + L\Gamma'(V(\rho,\phi)) + \tfrac{1}{2L} \\ &\text{and } \bar{\mathcal{M}}_2 \geq \tfrac{\rho-\rho_0}{h'(\rho)}\max\left\{1, \tfrac{\rho-\rho_0}{h'(\rho)\cos^2(\phi)}\right\} \text{ on } \mathcal{S} \end{aligned} \tag{19}$$

Curvature Identification (M-Sizemore-Zhang)

$$\begin{cases} \dot{\tilde{q}}_1 & = & -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 & = & h'(\tilde{q}_1+\rho_0)\cos(\tilde{q}_2) - \mu\sin(\tilde{q}_2) \\ & & -\frac{\tilde{\kappa}_0\cos(\phi)}{(1+\kappa_0(\rho-\rho_0))(1+(\kappa_0+\tilde{\kappa}_0)(\rho-\rho_0))} \\ \dot{\tilde{\kappa}}_0 & = & \frac{(\kappa_0+\tilde{\kappa}_0-\underline{c})(\bar{c}-\kappa_0-\tilde{\kappa}_0)}{(1+(\rho-\rho_0)\hat{\kappa}_0)^2}\cos(\tilde{q}_2)\frac{\partial U}{\partial \phi}(\rho,\phi). \end{cases} \tag{18}$$

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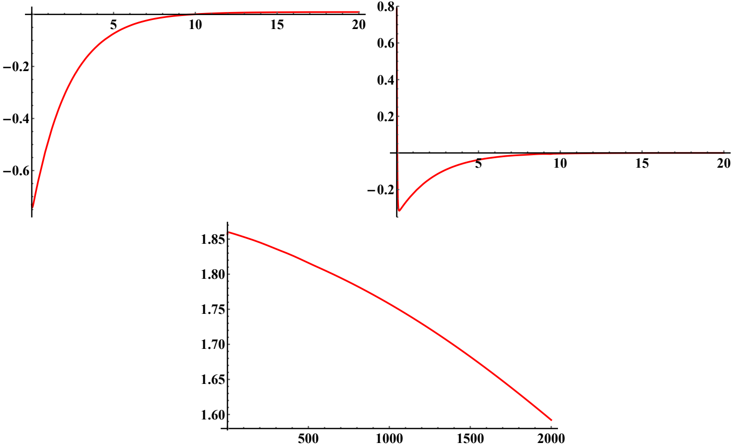
Theorem: If the bounds $\underline{c} \geq 0$ and $\bar{c} > 0$ on κ_0 satisfy

$$\bar{c} < \underline{c} + \min\left\{\frac{\bar{\Delta}}{4}, \frac{1}{2\sqrt{\bar{\mathcal{M}}_1}}, \frac{1}{2\sqrt{2\bar{\mathcal{M}}_2(2+\bar{\mathcal{M}}_1)}}\right\} \text{ and } \bar{c} < \frac{1}{2(\rho_0-\rho_*)} \tag{20}$$

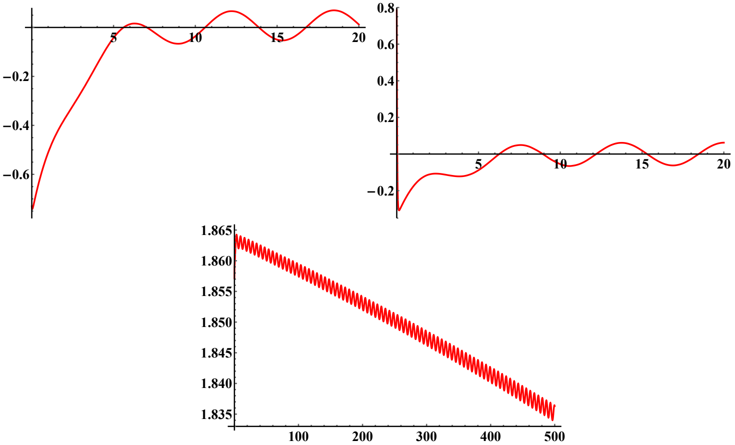
then (18) is GAS to 0 on $\{(\widetilde{q},\widetilde{\kappa}_0) : (\rho,\phi)\in\mathcal{S}, \hat{\kappa}_0 \in (\underline{c},\bar{c})\}$. $\square$

Plots of $\tilde{q}_1(t) = \rho(t) - \rho_0$, $\tilde{q}_2(t) = \phi(t)$, and $\hat{\kappa}(t)$



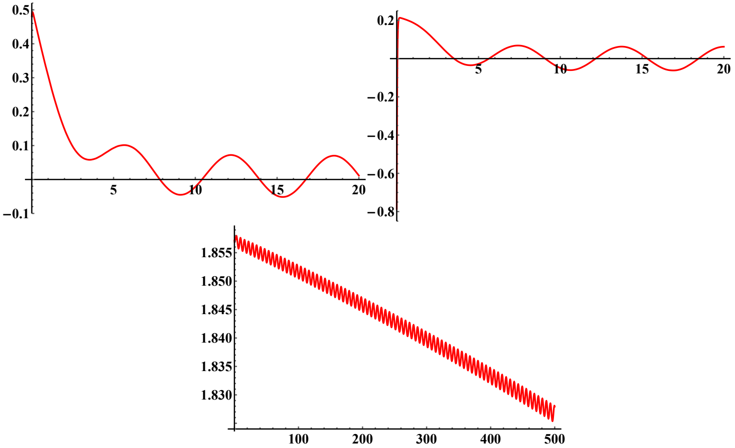
$\underline{c} = 0$, $\bar{c} = 2/3$

Plots of $\tilde{q}_1(t) = \rho(t) - \rho_0$, $\tilde{q}_2(t) = \phi(t)$, and $\hat{\kappa}(t)$

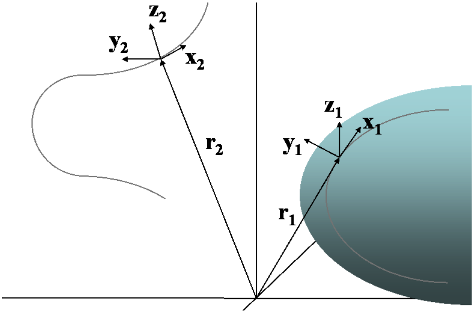


$\underline{c} = 0$, $\bar{c} = 2/3$, $\delta(t) = 2.5\sin(t)$ added to control

Plots of $\tilde{q}_1(t) = \rho(t) - \rho_0$, $\tilde{q}_2(t) = \phi(t)$, and $\hat{\kappa}(t)$



3D Curve Tracking by Unit Speed Robot



$$
\begin{aligned}
\dot{\mathbf{r}}_1 &= \alpha\, \mathbf{x_1} \\
\dot{\mathbf{x}}_1 &= \alpha\kappa_n\, \mathbf{y_1} + \alpha\kappa_g\, \mathbf{z_1} \\
\dot{\mathbf{y}}_1 &= -\alpha\kappa_n \mathbf{x_1} \\
\dot{\mathbf{z}}_1 &= -\alpha\kappa_g \mathbf{x_1}
\end{aligned}
$$

$$
\begin{aligned}
\dot{\mathbf{r}}_2 &= \mathbf{x_2} \\
\dot{\mathbf{x}}_2 &= u\mathbf{y_2} + v\mathbf{z_2} \\
\dot{\mathbf{y}}_2 &= -u\mathbf{x_2} \\
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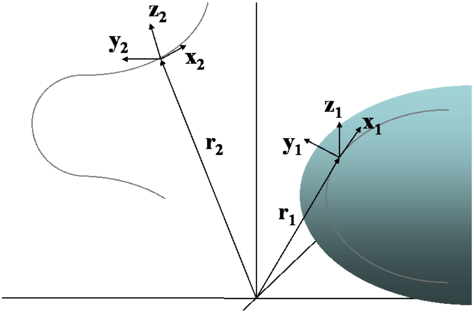

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Speed $\alpha = \mathrm{d}s/\mathrm{d}t \neq 0$. Controls: u and v. κ_n and κ_g are C^1 and nonpositive valued.

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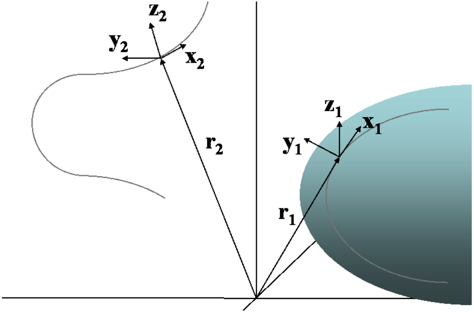

$$
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\dot{\mathbf{r}}_1 &= \alpha\,\mathbf{x}_1 \\
\dot{\mathbf{x}}_1 &= \alpha\kappa_n\,\mathbf{y}_1 + \alpha\kappa_g\,\mathbf{z}_1 \\
\dot{\mathbf{y}}_1 &= -\alpha\kappa_n\mathbf{x}_1 \\
\dot{\mathbf{z}}_1 &= -\alpha\kappa_g\mathbf{x}_1
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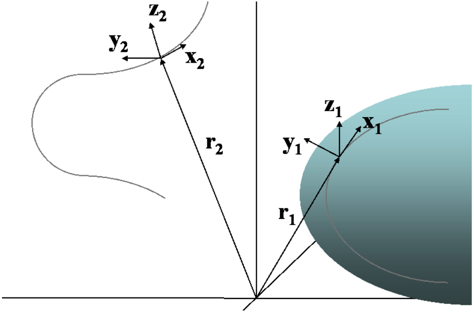

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\dot{\mathbf{r}}_1 &= \alpha \mathbf{x}_1 \\
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Speed $\alpha = \mathrm{d}s/\mathrm{d}t \neq 0$. Controls: u and v. κ_n and κ_g are C^1 and nonpositive valued. Zhang-Justh-Krishnaprasad CDC'04.

Goal: Find u and v such that $|\mathbf{r}_1(t) - \mathbf{r}_2(t)| \to \rho_c$ for a desired $\rho_c > 0$ and $\mathbf{x}_1 \cdot \mathbf{x}_2 \to 1$, while compensating for additive uncertainty and delays and identifying control gains.

Our New Variables and Control Design

$(\rho_1, \rho_2) = ((\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{y}_1, (\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{z}_1)$ has desired value (ρ_{c1}, ρ_{c2}).

$\rho_c = |(\rho_{c1}, \rho_{c2})|$. Shape vars: $\varphi = \mathbf{x}_1 \cdot \mathbf{x}_2$, $\beta = \mathbf{y}_1 \cdot \mathbf{x}_2$, $\gamma = \mathbf{z}_1 \cdot \mathbf{x}_2$

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$$
\begin{aligned}
u &= a_1(\mathbf{x}_1 \cdot \mathbf{y}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{y}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{y}_2),\\
v &= a_1(\mathbf{x}_1 \cdot \mathbf{z}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{z}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{z}_2),\\
a_1 &= \mu,\ a_2 = -h_1'(\rho_1) + \tfrac{\alpha\kappa_n}{\varphi},\ a_3 = -h_2'(\rho_2) + \tfrac{\alpha\kappa_g}{\varphi},\ \text{and} \qquad (21)\\
h_i(\rho_i) &= \begin{cases} \bar{c}\left(\rho_i + \rho_{ci}^2/\rho_i - 2\rho_{ci}\right), & \rho_i \in (0, \rho_{ci})\\ \frac{\bar{c}}{\rho_{ci}}(\rho_i - \rho_{ci})^2, & \rho_i \geq \rho_{ci} \end{cases}
\end{aligned}
$$

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$$\begin{aligned}
u &= a_1(\mathbf{x}_1 \cdot \mathbf{y}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{y}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{y}_2), \\
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\end{aligned}$$

New state $Y = (\rho_1, \zeta, \rho_2, \theta)$ takes its values in $\mathcal{X}^\sharp$, where $(\varphi, \beta, \gamma) = \big(\cos(\zeta)\cos(\theta), -\sin(\zeta)\cos(\theta), \sin(\theta)\big)$ and where $\mathcal{X}^\sharp = (0, \infty) \times (-\pi/2, \pi/2) \times (0, \infty) \times (-\pi/2, \pi/2)$.

A Key Ingredient: Strict Lyapunov Function

$$
\begin{aligned}
\dot{\rho}_1 &= -\sin(\zeta)\cos(\theta) \\
\dot{\zeta} &= -\frac{1}{\cos^2(\theta)}\Big[\alpha\kappa_n \sin^2(\theta) - h_1'(\rho_1)\cos(\zeta)\cos(\theta) \\
&\quad + \alpha\kappa_g \sin(\theta)\sin(\zeta)\cos(\theta) + \mu \sin(\zeta)\cos(\theta)\Big] \\
\dot{\rho}_2 &= \sin(\theta) \\
\dot{\theta} &= \alpha\kappa_g \frac{\sin^2(\zeta)}{\cos(\zeta)} - h_2'(\rho_2)\cos(\theta) - \mu\cos(\zeta)\sin(\theta) \\
&\quad + \left(-h_1'(\rho_1) + \frac{\alpha\kappa_n}{\cos(\theta)\cos(\zeta)}\right)\sin(\zeta)\sin(\theta)
\end{aligned}
\tag{22}
$$

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\dot{\zeta} &= -\frac{1}{\cos^2(\theta)}\Big[\alpha\kappa_n \sin^2(\theta) - h_1'(\rho_1)\cos(\zeta)\cos(\theta) \\
&\quad + \alpha\kappa_g \sin(\theta)\sin(\zeta)\cos(\theta) + \mu\sin(\zeta)\cos(\theta)\Big] \\
\dot{\rho}_2 &= \sin(\theta) \\
\dot{\theta} &= \alpha\kappa_g \frac{\sin^2(\zeta)}{\cos(\zeta)} - h_2'(\rho_2)\cos(\theta) - \mu\cos(\zeta)\sin(\theta) \\
&\quad + \left(-h_1'(\rho_1) + \frac{\alpha\kappa_n}{\cos(\theta)\cos(\zeta)}\right)\sin(\zeta)\sin(\theta)
\end{aligned}
\tag{22}
$$

Theorem (MZ, SICON'15): We can build a function $\mathcal{L}$ such that

$$U(Y) = -h_1'(\rho_1)\sin(\zeta)\cos(\theta) + h_2'(\rho_2)\sin(\theta) + \int_0^{V(Y)} \mathcal{L}(q)\mathrm{d}q$$

is a strict Lyapunov function for (22) on $\mathcal{X}^\sharp$ for the equilibrium $(\rho_{c1}, 0, \rho_{c2}, 0)$, where $V(Y) = -\ln(\cos(\theta)\cos(\zeta)) + h_1(\rho_1) + h_2(\rho_2)$.

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Our hexagons ensure desirable tolerance and safety bounds.

Challenges remain for other systems where P enters nonlinearly.

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