Designs and Theory for State-Constrained Nonlinear Feedback Controls for Delay Systems: An Infomercial

Michael Malisoff, Roy P. Daniels Professor of Mathematics
Louisiana State University

JOINT WITH ENGINEERING AND MATHEMATICS COLLEAGUES AND STUDENTS

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Applied Analysis Seminar
LSU Department of Mathematics
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Michael Malisoff awarded two collaborative research grants from the NSF Directorate for Engineering

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Roy Paul Daniels Professor **Michael Malisoff** will serve as Lead Principal Investigator for two collaborative projects from the US National Science Foundation Directorate for Engineering. The first project "**Robustness of Networked Model Predictive Control Satisfying Critical Timing Constraints**," focuses on resolving contentions in a class of communication networks that are common in automobiles and other real-time control applications, and is joint with the Georgia Institute of Technology School of Electrical and Computer Engineering. The second project is "**Designs and Theory of State-Constrained Nonlinear Feedback Controls for Delay and Partial Differential Equation Systems**" and covers control designs for classes of ordinary and hyperbolic partial differential equations that arise in oil production and rehabilitation engineering, and is joint with the UC San Diego Department of Mechanical and Aerospace Engineering.

Print

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Collaborators: Fumin Zhang from Georgia Tech and Miroslav Krstic (MK) from UCSD

Current PhD Students: Ruzhou Yang, Ningshi Yao

Total 3-Year Budget: $871,000 (LSU Portion: $380,928)

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These are *doubly* parameterized families of ODEs of the form

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$$Y'(t) = \mathcal{G}(t, Y_t, \delta(t)), \quad Y(t) \in \mathcal{Y}, \qquad (2)$$

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Typically we construct u such that all trajectories of (2) for all possible choices of δ satisfy some control objective.

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Find γ_i's by building certain LKFs for $Y'(t) = \mathcal{G}(t, Y_t, 0)$.

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$$V^\sharp(Y_t) = V(Y(t)) + \frac{1}{4}\int_{t-\tau}^{t}|Y(\ell)|^2\mathrm{d}\ell + \frac{1}{8\tau}\int_{t-\tau}^{t}\left[\int_s^t |Y(r)|^2\mathrm{d}r\right]\mathrm{d}s$$

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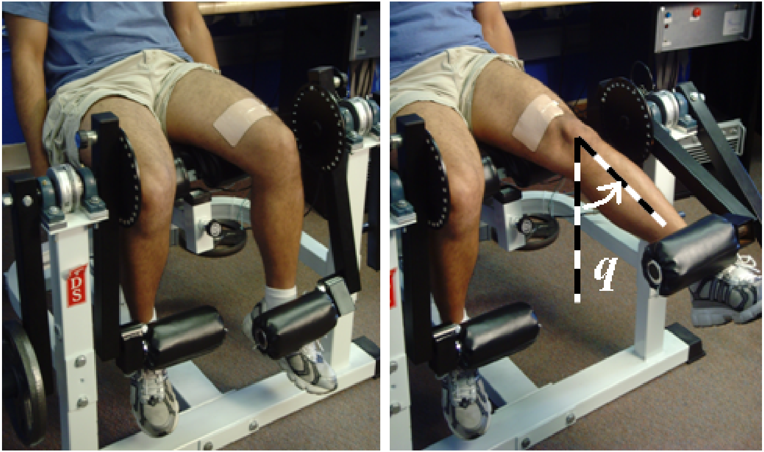
Our new control only needs sampled observations, allows any delay, and tracks position and velocity under a state constraint.

NMES on Leg Extension Machine

(Loading Video...)

Leg extension machine at Warren Dixon's NCR Lab at U of FL

NMES on Leg Extension Machine



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Mathematical Model

$$\overbrace{J\ddot{q}}^{M_I(\ddot{q})} + \overbrace{b_1\dot{q} + b_2\tanh(b_3\dot{q})}^{M_V(\dot{q})} + \overbrace{k_1 q e^{-k_2 q} + k_3\tan(q)}^{M_e(q)} + \underbrace{\mathcal{M} g l \sin(q)}_{M_g(q)} = \mathcal{A}(q,\dot{q})\, v(t-\tau), \quad q \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right) \tag{3}$$

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J and $\mathcal{M}$ are inertia and mass of the lower limb/machine, the b_i's and k_i's are positive damping and elastic constants, respectively, l is the distance between the knee joint and the center of the mass of the lower limb/machine, $\tau > 0$ is a delay.

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$$\max\{||\dot{q}_d||_\infty, ||v_d||_\infty, ||\dot{v}_d||_\infty\} < \infty \text{ and } ||q_d||_\infty < \tfrac{\pi}{2} \tag{6}$$

Voltage Potential Controller

$$v(t) = \frac{g_2(\zeta_d(t+\tau))v_d(t) - g_1(\zeta_d(t+\tau)+\xi(t)) + g_1(\zeta_d(t+\tau)) - (1+\mu^2)\xi_1(t) - 2\mu\xi_2(t)}{g_2(\zeta_d(t+\tau)+\xi(t))}$$

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and $\xi(T_i) = z_{N_i}$.

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and $\xi(T_i) = z_{N_i}$. The time-varying Euler iterations $\{z_k\}$ at each time T_i use measurements $(q(T_i), \dot{q}(T_i))$.

Voltage Potential Controller (continued)

Euler iterations used for control:

$$z_{k+1} = \Omega(T_i + kh_i, h_i, z_k; v) \ \text{for} \ k = 0, ..., N_i - 1 \ , \ \text{where}$$

$$z_0 = \begin{pmatrix} \tan(q(T_i)) - \tan(q_d(T_i)) \\ \frac{\dot{q}(T_i)}{\cos^2(q(T_i))} - \frac{\dot{q}_d(T_i)}{\cos^2(q_d(T_i))} \end{pmatrix}, \quad h_i = \frac{\tau}{N_i} \ ,$$

and $\Omega : [0, +\infty)^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$\Omega(T, h, x; v) = \begin{bmatrix} \Omega_1(T, h, x; v) \\ \Omega_2(T, h, x; v) \end{bmatrix} \tag{7}$$

and the formulas

$$\begin{aligned} \Omega_1(T, h, x; v) &= x_1 + hx_2 \ \text{and} \\ \Omega_2(T, h, x; v) &= x_2 + \zeta_{2,d}(T) + \int_T^{T+h} g_1(\zeta_d(s) + x) \mathrm{d}s \\ &\quad + \int_T^{T+h} g_2(\zeta_d(s) + x)\, v(s - \tau) \mathrm{d}s - \zeta_{2,d}(T + h). \end{aligned}$$

NMES Theorem (IK, MM, MK, Ruzhou, IJRNC)

For all positive constants τ and r, there exist a locally bounded function N, a constant $\omega \in (0, \mu/2)$ and a locally Lipschitz function C satisfying $C(0) = 0$ such that: For all sample times $\{T_i\}$ in $[0, \infty)$ such that $\sup_{i\geq 0}(T_{i+1} - T_i) \leq r$ and each initial condition, the solution $(q(t), \dot{q}(t), v(t))$ with

$$N_i = N\left(\left|\left(\tan(q(T_i)), \frac{\dot{q}(T_i)}{\cos^2(q(T_i))}\right) - \zeta_d(T_i)\right| + ||v - v_d||_{[T_i-\tau, T_i]}\right) \quad (8)$$

satisfies

$$\begin{aligned} &|q(t) - q_d(t)| + |\dot{q}(t) - \dot{q}_d(t)| + ||v - v_d||_{[t-\tau,t]} \\ \leq\ & e^{-\omega t} C\left(\frac{|q(0)-q_d(0)|+|\dot{q}(0)-\dot{q}_d(0)|}{\cos^2(q(0))} + ||v_0 - v_d||_{[-\tau,0]}\right) \end{aligned}$$

for all $t \geq 0$.

First Simulation

$$J\ddot{q} + b_1\dot{q} + b_2 \tanh(b_3\dot{q}) + k_1 q e^{-k_2 q} + k_3 \tan(q) + \mathcal{M}gl \sin(q) = \mathcal{A}(q,\dot{q})\, v(t-\tau), \quad q \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right) \tag{9}$$

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$\tau = 0.07\mathrm{s}$, $\mathcal{A}(q,\dot{q}) = \bar{a}e^{-2q^2}\sin(q) + \bar{b}$

$$\begin{aligned} &J = 0.39\,\mathrm{kg\text{-}m^2/rad},\ b_1 = 0.6\,\mathrm{kg\text{-}m^2/(rad\text{-}s)},\ \bar{a} = 0.058, \\ &b_2 = 0.1\,\mathrm{kg\text{-}m^2/(rad\text{-}s)},\ b_3 = 50\,\mathrm{s/rad},\ \bar{b} = 0.0284, \\ &k_1 = 7.9\,\mathrm{kg\text{-}m^2/(rad\text{-}s^2)},\ k_2 = 1.681/\mathrm{rad}, \\ &k_3 = 1.17\,\mathrm{kg\text{-}m^2/(rad\text{-}s^2)},\ \mathcal{M} = 4.38\,\mathrm{kg},\ l = 0.248\,\mathrm{m}. \end{aligned} \tag{10}$$

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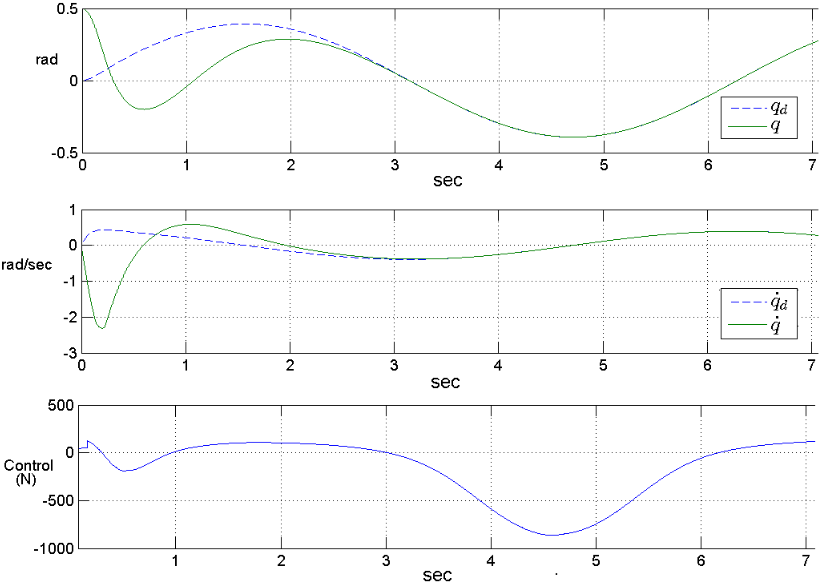
$q(0) = 0.5$ rad, $\dot{q}(0) = 0$ rad/s, $v(t) = 0$ on $[-0.07, 0)$, $N_i = N = 10$, and $T_{i+1} - T_i = 0.014$s, and $\mu = 2$.

First Simulation



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We took $\tau = 0.07$s and the same model parameters

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$$q_d(t) = \frac{\pi}{3}\left(1 - \exp(-3t)\right)\ \mathrm{rad}, \tag{13}$$

$q(0) = \frac{\pi}{18}$, $\dot{q}(0) = v_0(t) = 0$, $N_i = N = 10$, $T_{i+1} - T_i = 0.014$.

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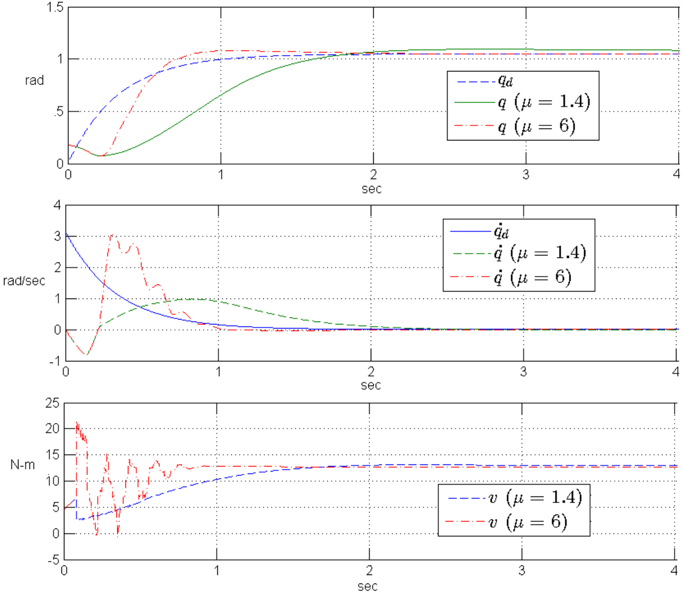
We used these mismatched parameters in the control:

$$\begin{aligned} &J' = 1.25J,\ \ b_1' = 1.2b_1,\ \ b_2' = 0.9b_2,\ \bar{a}' = 1.185\bar{a}, \\ &b_3' = 0.85b_3,\ \ k_1' = 1.1k_1,\ \ k_2' = 0.912k_2,\ \bar{b}' = 0.98\bar{b}, \\ &k_3' = 0.9k_3,\ \ \mathcal{M}' = 0.97\mathcal{M},\ \ \text{and}\ \ l' = 1.013l. \end{aligned} \tag{14}$$

Simulated Robustness Test



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In future work, we hope to apply input-to-state stability to better understand the effects of uncertainties under state constraints.

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Even though my projects are sponsored by NSF Engineering, you do not need any engineering background to be considered.

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It uses event-triggered controls and robust forward invariance to help determine how much uncertainty a system can tolerate.

Openings for PhD Students (cont'd)

malisoff@lsu.edu, 225-578-6714