Control and Robustness Analysis for Curve Tracking with Unknown Control Gains

Michael Malisoff, Roy P. Daniels Professor
Louisiana State University Department of Mathematics

Joint with Fumin Zhang from Georgia Tech School of ECE
Sponsored by NSF/ECCS/EPAS Program

Summary of Forthcoming Paper in Automatica

2013 Joint Mathematics Meetings
Special Session on Theory and Interdisciplinary Applications of Dynamical Systems
January 9, 2013

What Are Perturbed Control Systems?

What Are Perturbed Control Systems ?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \tag{1}$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.
The functions $\delta : [0, \infty) \to \mathcal{D}$ represent uncertainty.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \tag{1}$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.
The functions $\delta : [0, \infty) \to \mathcal{D}$ represent uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.
The functions $\delta : [0, \infty) \to \mathcal{D}$ represent uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
The vector Γ is constant but unknown.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.
The functions $\delta : [0, \infty) \to \mathcal{D}$ represent uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
The vector Γ is constant but unknown.

Specify u to get a *doubly* parameterized closed loop family

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}, \qquad (2)$$

where $\mathcal{G}(t, Y, \Gamma, d) = \mathcal{F}(t, Y, u(t, Y), \Gamma, d)$.

What Are Perturbed Control Systems?

These are *triply* parameterized families of ODEs of the form

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \tag{1}$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. We have freedom to choose the control function u.
The functions $\delta : [0, \infty) \to \mathcal{D}$ represent uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
The vector Γ is constant but unknown.

Specify u to get a *doubly* parameterized closed loop family

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}, \tag{2}$$

where $\mathcal{G}(t, Y, \Gamma, d) = \mathcal{F}(t, Y, u(t, Y), \Gamma, d)$.

Typically we construct u such that all trajectories of (2) for all possible choices of δ satisfy some control objective.

What is One Possible Control Objective?

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded.

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

$$Y'(t) = \mathcal{G}\big(t, Y(t), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma_{\rm pert})$$

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

$$Y'(t) = \mathcal{G}\big(t, Y(t), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma_{\text{pert}})$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) + \gamma_3(|\delta|_{[t_0,t]}) \qquad \text{(ISS)}$$

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

$$Y'(t) = \mathcal{G}\big(t, Y(t), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma_{\rm pert})$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) + \gamma_3(|\delta|_{[t_0,t]}) \qquad \text{(ISS)}$$

Find γ_i's by building special strict LFs for $Y'(t) = \mathcal{G}(t, Y(t), \Gamma, 0)$.

What is One Possible Control Objective?

Input-to-state stability generalizes global asymptotic stability.

$$Y'(t) = \mathcal{G}(t, Y(t), \Gamma), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma)$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) \qquad \text{(UGAS)}$$

Our γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

$$Y'(t) = \mathcal{G}\big(t, Y(t), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y} \qquad (\Sigma_{\rm pert})$$

$$|Y(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y(t_0)|)\right) + \gamma_3(|\delta|_{[t_0,t]}) \qquad \text{(ISS)}$$

Find γ_i's by building special strict LFs for $Y'(t) = \mathcal{G}(t, Y(t), \Gamma, 0)$.

Ex : $\Sigma_{\rm pert}$ is ISS iff it has an ISS Lyapunov function (Sontag-Wang)

Adaptive Tracking and Parameter Identification

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} = \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R.

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} = \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R.
That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} = \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R.
That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem:

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} \;=\; \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R.
That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem: Find a dynamic feedback and a parameter estimator

$$u(t, \xi, \hat{\Gamma}) \;\; \text{and} \;\; \dot{\hat{\Gamma}} \;=\; \tau(t, \xi, \hat{\Gamma}) \tag{4}$$

that makes the $Y = (\tilde{\xi}, \tilde{\Gamma}) = (\xi - \xi_R, \hat{\Gamma} - \Gamma)$ dynamics ISS.

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} = \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R. That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem: Find a dynamic feedback and a parameter estimator

$$u(t, \xi, \hat{\Gamma}) \ \text{ and } \ \dot{\hat{\Gamma}} = \tau(t, \xi, \hat{\Gamma}) \tag{4}$$

that makes the $Y = (\tilde{\xi}, \tilde{\Gamma}) = (\xi - \xi_R, \hat{\Gamma} - \Gamma)$ dynamics ISS.

Flight control, electrical and mechanical engineering, etc.

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} \;=\; \mathcal{J}(t, \xi, \Gamma, u, \delta) \qquad (3)$$

with a smooth reference trajectory ξ_R for a reference control u_R. That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem: Find a dynamic feedback and a parameter estimator

$$u(t, \xi, \hat{\Gamma}) \;\; \text{and} \;\; \dot{\hat{\Gamma}} \;=\; \tau(t, \xi, \hat{\Gamma}) \qquad (4)$$

that makes the $Y = (\tilde{\xi}, \tilde{\Gamma}) = (\xi - \xi_R, \hat{\Gamma} - \Gamma)$ dynamics ISS.

Flight control, electrical and mechanical engineering, etc.
Persistent excitation.

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} \;=\; \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R.
That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem: Find a dynamic feedback and a parameter estimator

$$u(t, \xi, \hat{\Gamma}) \;\; \text{and} \;\; \dot{\hat{\Gamma}} \;=\; \tau(t, \xi, \hat{\Gamma}) \tag{4}$$

that makes the $Y = (\tilde{\xi}, \tilde{\Gamma}) = (\xi - \xi_R, \hat{\Gamma} - \Gamma)$ dynamics ISS.

Flight control, electrical and mechanical engineering, etc.
Persistent excitation. Annaswamy, Narendra, Teel,..

Adaptive Tracking and Parameter Identification

Consider a perturbed control system

$$\dot{\xi} = \mathcal{J}(t, \xi, \Gamma, u, \delta) \tag{3}$$

with a smooth reference trajectory ξ_R for a reference control u_R. That means $\xi_R'(t) = \mathcal{J}(t, \xi_R(t), \Gamma, u_R(t), 0)\ \forall t \geq 0$.

Problem: Find a dynamic feedback and a parameter estimator

$$u(t, \xi, \hat{\Gamma}) \ \text{ and } \ \dot{\hat{\Gamma}} = \tau(t, \xi, \hat{\Gamma}) \tag{4}$$

that makes the $Y = (\tilde{\xi}, \tilde{\Gamma}) = (\xi - \xi_R, \hat{\Gamma} - \Gamma)$ dynamics ISS.

Flight control, electrical and mechanical engineering, etc.
Persistent excitation. Annaswamy, Narendra, Teel,..

We proved a general theorem about how this can be solved.

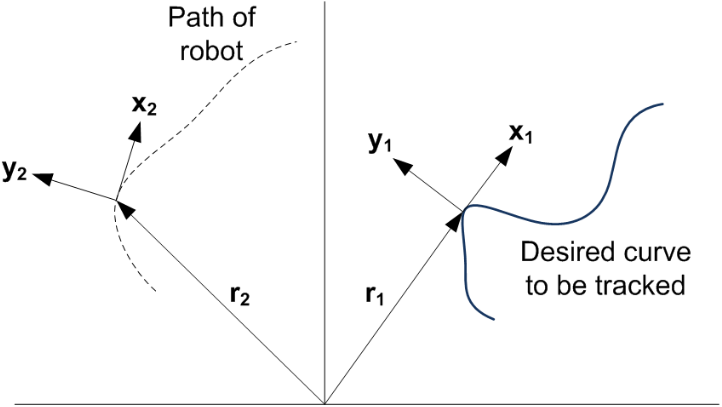
Application: 2D Curve Tracking for Marine Robots

Application: 2D Curve Tracking for Marine Robots

Motivation: Search for pollutants from Deepwater Horizon disaster.

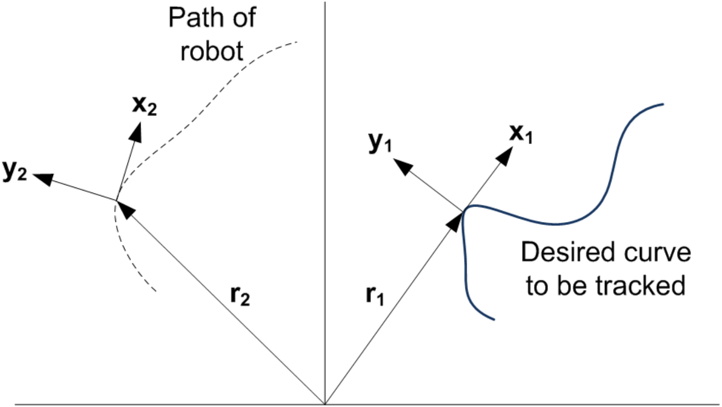
Application: 2D Curve Tracking for Marine Robots

Motivation: Search for pollutants from Deepwater Horizon disaster.



Application: 2D Curve Tracking for Marine Robots

Motivation: Search for pollutants from Deepwater Horizon disaster.



$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

Adaptive Robust Curve Tracking

Adaptive Robust Curve Tracking

$$\begin{cases} \dot{\rho} &= -\sin(\phi) \\ \dot{\phi} &= \frac{\kappa\cos(\phi)}{1+\kappa\rho} + \Gamma[u+\delta] \end{cases} \quad (\rho,\phi) \in \overbrace{(0,\infty)\times(-\pi/2,\pi/2)}^{\text{state space}} \quad (\Sigma_c)$$

Adaptive Robust Curve Tracking

$$\left\{\begin{array}{lcl}\dot{\rho} & = & -\sin(\phi) \\ \dot{\phi} & = & \frac{\kappa\cos(\phi)}{1+\kappa\rho} + \Gamma[u+\delta]\end{array}\right. \quad (\rho,\phi) \in \overbrace{(0,\infty)\times(-\pi/2,\pi/2)}^{\text{state space}} \quad (\Sigma_c)$$

$$h(\rho) = \alpha\left\{\rho + \frac{\rho_0^2}{\rho} - 2\rho_0\right\}, \quad \rho_0 = \text{desired value for } \rho \qquad (5)$$

Adaptive Robust Curve Tracking

$$\begin{cases} \dot{\rho} &= -\sin(\phi) \\ \dot{\phi} &= \frac{\kappa\cos(\phi)}{1+\kappa\rho} + \Gamma[u+\delta] \end{cases} \quad (\rho,\phi) \in \overbrace{(0,\infty)\times(-\pi/2,\pi/2)}^{\text{state space}} \qquad (\Sigma_c)$$

$$h(\rho) = \alpha\left\{\rho + \frac{\rho_0^2}{\rho} - 2\rho_0\right\}, \quad \rho_0 = \text{desired value for } \rho \qquad (5)$$

$$\text{Control}: \quad u(\rho,\phi,\hat{\Gamma}) = -\frac{1}{\hat{\Gamma}}\left(\frac{\kappa\cos(\phi)}{1+\kappa\rho} - h'(\rho)\cos(\phi) + \mu\sin(\phi)\right) \qquad (6)$$

$$\text{Estimator}: \quad \dot{\widehat{\Gamma}} = (\widehat{\Gamma} - c_{\min})(c_{\max} - \widehat{\Gamma})\frac{\partial V^\sharp(\rho,\phi)}{\partial\phi}u(\rho,\phi,\hat{\Gamma}) \qquad (7)$$

Adaptive Robust Curve Tracking

$$\begin{cases} \dot{\rho} & = & -\sin(\phi) \\ \dot{\phi} & = & \frac{\kappa\cos(\phi)}{1+\kappa\rho} + \Gamma[u+\delta] \end{cases} \quad (\rho,\phi) \in \overbrace{(0,\infty)\times(-\pi/2,\pi/2)}^{\text{state space}} \quad (\Sigma_c)$$

$$h(\rho) = \alpha\left\{\rho + \frac{\rho_0^2}{\rho} - 2\rho_0\right\}, \quad \rho_0 = \text{desired value for } \rho \qquad (5)$$

$$\text{Control}: \quad u(\rho,\phi,\hat{\Gamma}) = -\frac{1}{\hat{\Gamma}}\left(\frac{\kappa\cos(\phi)}{1+\kappa\rho} - h'(\rho)\cos(\phi) + \mu\sin(\phi)\right) \qquad (6)$$

$$\text{Estimator}: \quad \dot{\hat{\Gamma}} = (\widehat{\Gamma} - c_{\min})(c_{\max} - \widehat{\Gamma})\frac{\partial V^\sharp(\rho,\phi)}{\partial\phi} u(\rho,\phi,\hat{\Gamma}) \qquad (7)$$

$$V^\sharp(\rho,\phi) = -h'(\rho)\sin(\phi) + \int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m \qquad (8)$$

$$\gamma(q) = \frac{1}{\mu}\left(\frac{2}{\alpha^2\rho_0^4}(q+2\alpha\rho_0)^3 + 1\right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4\alpha^2}q^3 \qquad (9)$$

$$V(\rho,\phi) = -\ln\left(\cos(\phi)\right) + h(\rho) \qquad (10)$$

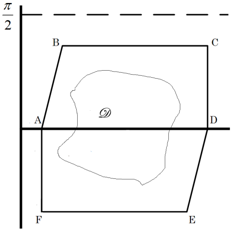
Robustly Forwardly Invariant Hexagonal Regions

Robustly Forwardly Invariant Hexagonal Regions

We must restrict the suprema of the perturbations $\delta(t)$ to keep (ρ, ϕ) from exiting the required state space $(0, \infty) \times (-\pi/2, \pi/2)$.

Robustly Forwardly Invariant Hexagonal Regions

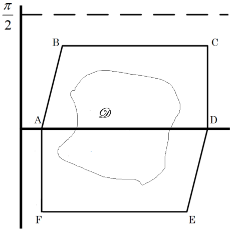
We must restrict the suprema of the perturbations $\delta(t)$ to keep (ρ, ϕ) from exiting the required state space $(0, \infty) \times (-\pi/2, \pi/2)$.



View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$.

Robustly Forwardly Invariant Hexagonal Regions

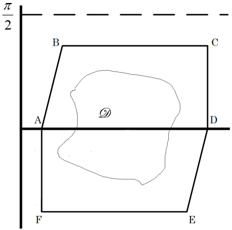
We must restrict the suprema of the perturbations $\delta(t)$ to keep (ρ, ϕ) from exiting the required state space $(0, \infty) \times (-\pi/2, \pi/2)$.



View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$. For each i, all trajectories of (Σ_c) starting in H_i for all $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$ stay in H_i. The tilted legs have slope $c_{\min}\mu / c_{\max}$.

Robustly Forwardly Invariant Hexagonal Regions

We must restrict the suprema of the perturbations $\delta(t)$ to keep (ρ, ϕ) from exiting the required state space $(0, \infty) \times (-\pi/2, \pi/2)$.

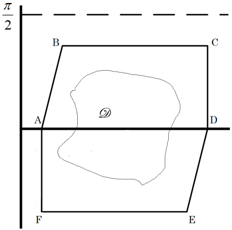


View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$. For each i, all trajectories of (Σ_c) starting in H_i for all $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$ stay in H_i. The tilted legs have slope $c_{\min}\mu/c_{\max}$.

For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Robustly Forwardly Invariant Hexagonal Regions

We must restrict the suprema of the perturbations $\delta(t)$ to keep (ρ, ϕ) from exiting the required state space $(0, \infty) \times (-\pi/2, \pi/2)$.



View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$. For each i, all trajectories of (Σ_c) starting in H_i for all $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$ stay in H_i. The tilted legs have slope $c_{\text{min}}\mu/c_{\max}$.

For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Then we can prove ISS of the tracking and parameter identification dynamics for each set H_i for the disturbance set $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

Summary (M. and Zhang, Automatica, 2013)

Summary (M. and Zhang, Automatica, 2013)

Theorem

Summary (M. and Zhang, Automatica, 2013)

Theorem

Let $\kappa > 0$ *and* $\rho_0 > 0$ *be any constants.*

Summary (M. and Zhang, Automatica, 2013)

Theorem

Let $\kappa > 0$ *and* $\rho_0 > 0$ *be any constants. Let* $c_{\min}$ *and* $c_{\max}$ *be any positive constants such that* $c_{\min} < \Gamma < c_{\max}$.

Summary (M. and Zhang, Automatica, 2013)

Theorem

Let $\kappa > 0$ *and* $\rho_0 > 0$ *be any constants. Let* $c_{\min}$ *and* $c_{\max}$ *be any positive constants such that* $c_{\min} < \Gamma < c_{\max}$*. Let* $i \in \mathbb{N}$*, and let* H_i *and* δ_{*i} *satisfy the above requirements.*

Summary (M. and Zhang, Automatica, 2013)

Theorem

Let $\kappa > 0$ and $\rho_0 > 0$ be any constants. Let $c_{\min}$ and $c_{\max}$ be any positive constants such that $c_{\min} < \Gamma < c_{\max}$. Let $i \in \mathbb{N}$, and let H_i and δ_{*i} satisfy the above requirements. Then the augmented perturbed 2D tracking and parameter identification dynamics

$$\begin{cases} \dot{Y}_1 &= -\sin(Y_2) \\ \dot{Y}_2 &= \frac{\kappa\cos(Y_2)}{1+\kappa(Y_1+\rho_0)} + \Gamma u\left(Y_1+\rho_0, Y_2, \hat{\Gamma}\right) + \Gamma\delta \\ \dot{\tilde{\Gamma}} &= u\left(Y_1+\rho_0, Y_2, \hat{\Gamma}\right)\left(\hat{\Gamma} - c_{\min}\right)\left(c_{\max} - \hat{\Gamma}\right)\frac{\partial V^\sharp(Y_1+\rho_0, Y_2)}{\partial Y_2} \end{cases}$$

for $Y = (Y_1, Y_2, \tilde{\Gamma}) = (\rho - \rho_0, \phi, \hat{\Gamma} - \Gamma)$ is ISS on the state space $\mathcal{Y} = (H_i - \{(\rho_0, 0)\}) \times (c_{\min} - \Gamma, c_{\max} - \Gamma)$ for $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

Summary (M. and Zhang, Automatica, 2013)

Theorem

Let $\kappa > 0$ and $\rho_0 > 0$ be any constants. Let $c_{\min}$ and $c_{\max}$ be any positive constants such that $c_{\min} < \Gamma < c_{\max}$. Let $i \in \mathbb{N}$, and let H_i and δ_{*i} satisfy the above requirements. Then the augmented perturbed 2D tracking and parameter identification dynamics

$$\begin{cases} \dot{Y}_1 = -\sin(Y_2) \\ \dot{Y}_2 = \frac{\kappa\cos(Y_2)}{1+\kappa(Y_1+\rho_0)} + \Gamma u\left(Y_1+\rho_0, Y_2, \hat{\Gamma}\right) + \Gamma\delta \\ \dot{\tilde{\Gamma}} = u\left(Y_1+\rho_0, Y_2, \hat{\Gamma}\right)\left(\hat{\Gamma} - c_{\min}\right)\left(c_{\max} - \hat{\Gamma}\right)\frac{\partial V^\sharp(Y_1+\rho_0, Y_2)}{\partial Y_2} \end{cases}$$

for $Y = (Y_1, Y_2, \tilde{\Gamma}) = (\rho - \rho_0, \phi, \hat{\Gamma} - \Gamma)$ is ISS on the state space $\mathcal{Y} = (H_i - \{(\rho_0, 0)\}) \times (c_{\min} - \Gamma, c_{\max} - \Gamma)$ for $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

Conclusions

Conclusions

- It is important but complicated to design controllers when there are unknown parameters that we must identify.

Conclusions

- It is important but complicated to design controllers when there are unknown parameters that we must identify.
- We overcame this challenge for an interesting large class of dynamics, including 2D curve tracking dynamics.

Conclusions

- It is important but complicated to design controllers when there are unknown parameters that we must identify.
- We overcame this challenge for an interesting large class of dynamics, including 2D curve tracking dynamics.
- Our robust forward invariance approach leads to input-to-state stability under maximal perturbation bounds.

Conclusions

- It is important but complicated to design controllers when there are unknown parameters that we must identify.
- We overcame this challenge for an interesting large class of dynamics, including 2D curve tracking dynamics.
- Our robust forward invariance approach leads to input-to-state stability under maximal perturbation bounds.
- We can also cover time delayed perturbed systems which model intermittent communication in marine environments.

Conclusions

- It is important but complicated to design controllers when there are unknown parameters that we must identify.
- We overcame this challenge for an interesting large class of dynamics, including 2D curve tracking dynamics.
- Our robust forward invariance approach leads to input-to-state stability under maximal perturbation bounds.
- We can also cover time delayed perturbed systems which model intermittent communication in marine environments.
- We can generalize our work to 3D curve tracking and we plan extensions to cases with other obstacles.