

# Adaptive Tracking and Parameter Identification

Michael Malisoff

## Basic Problem Formulation

Consider a system of differential equations

$$\dot{\xi} = f(\xi, P, u) \quad (1)$$

with a vector  $P$  of **unknown constant** parameters and functions  $\xi_R$  and  $u_R$  such that  $\dot{\xi}_R(t) = f(\xi_R(t), P, u_R(t))$  for all  $t \geq 0$ .



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such that with the control choice  $u(\xi, \hat{P})$  in (1), all solutions  $Y = (\tilde{\xi}, \tilde{P}) = (\xi - \xi_R, P - \hat{P})$  converge to 0 as  $t \rightarrow +\infty$ .

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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...

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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...

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Flight control, mechanical systems, robotics,...

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ZP. Jiang, E. Justh, P. Krishnaprasad, V. Lumelsky, A. Stepanov

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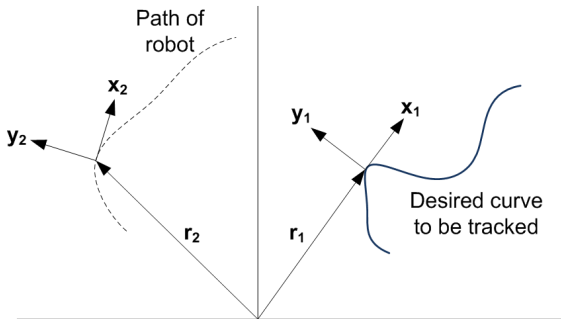
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Strict decay: there is a continuous positive definite  $\alpha$  such that  $\frac{d}{dt} V(Y(t)) \leq -\alpha(Y(t))$  along all solutions of system.

# Gyroscopic Model (with Georgia Tech)

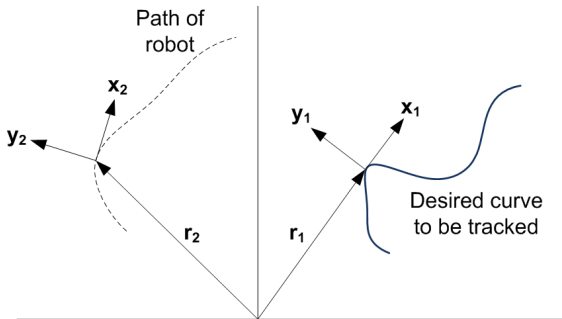
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Simpler 2D case: Boundary following with gyroscopic control.



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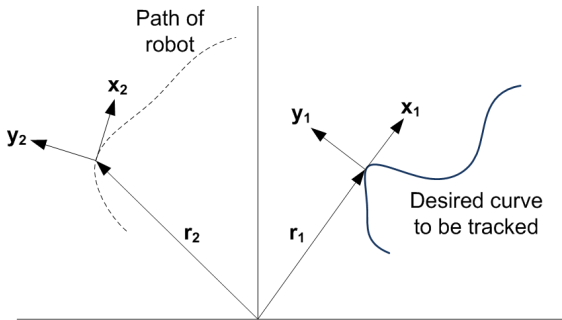
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Zhang-Justh-Krishnaprasad, IEEE-CDC'04.

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$$\rho = |\mathbf{r}_2 - \mathbf{r}_1|, \phi = \text{angle between } \mathbf{x}_1 \text{ and } \mathbf{x}_2, \cos(\phi) = \mathbf{x}_1 \cdot \mathbf{x}_2$$

# Curve Tracking Dynamics for 2D

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Along all solutions of (CL) for all  $t \geq 0$ , we have  $\frac{d}{dt} V(\rho, \phi) \leq 0$ .

## Strict Lyapunov Function (Mazenc-M-Z, TAC)

**Theorem 1:** The closed loop system (CL) has the strict Lyapunov function

$$U(Y) = -h'(\rho) \sin(\phi) + \frac{1}{\mu} \int_0^{V(\rho, \phi)} \gamma(m) dm + \Gamma(V(\rho, \phi)) + V(\rho, \phi),$$

where  $\gamma(q) = \frac{2(q+2\rho_0)^3}{\rho_0^4} + 1 + 0.5\mu^2 + \mu$ ,  $Y = (\rho - \rho_0, \phi)$ ,

$$\Gamma(q) = \frac{18}{\rho_0} q + 9 \left( \frac{2}{\rho_0} \right)^4 q^4, \text{ and } V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho)$$

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$$U(Y(t)) \geq V(\rho(t), \phi(t)) \quad (\text{PD})$$

$$\frac{d}{dt} U(Y(t)) \leq -0.5[h'(\rho(t)) \cos(\phi(t))]^2 - \sin^2(\phi(t)) \quad (\text{SD})$$

## Unknown Control Gains (M-Zhang)

$$\left\{ \begin{array}{l} \dot{\rho} = -\sin(\phi) \\ \dot{\phi} = \frac{\kappa \cos(\phi)}{1+\kappa\rho} + Ku, \quad K \in (c_{\min}, c_{\max}) \subseteq (0, \infty) \\ \dot{\hat{K}} = (\hat{K} - c_{\min})(c_{\max} - \hat{K}) \frac{\partial U}{\partial \phi} u, \quad \hat{K} \in (c_{\min}, c_{\max}) \end{array} \right. \quad (7)$$



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## Field Work at Grand Isle, LA



20 days of field work off Grand Isle. Search for oil spill remnants.  
Georgia Tech Savannah Robotics (co-led by Fumin Zhang)

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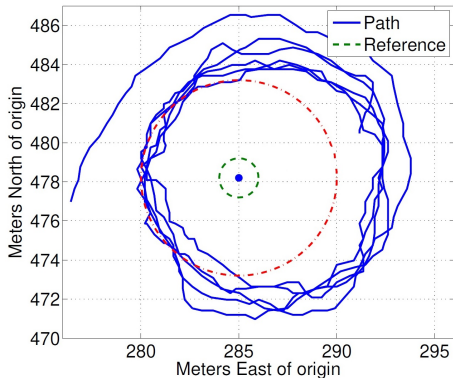
The slide features a blue and white abstract background. At the top center, the name "Victoria" is displayed in a dark blue box. To the right is a logo for "GTSR" featuring a stylized yellow and black bee. Below the name is a photograph of the Victoria, a small white autonomous surface vehicle with a black mast and sensor equipment. To the right of the photo is a "Vehicle Info" table.

Vehicle Info	
Vehicle Type	Surface
Operated	Remote & Autonomous
Weight (lbs)	110
Launched	May 2010

A play button icon is located in the bottom left corner of the slide.

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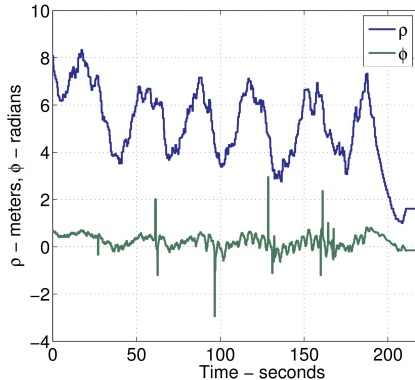
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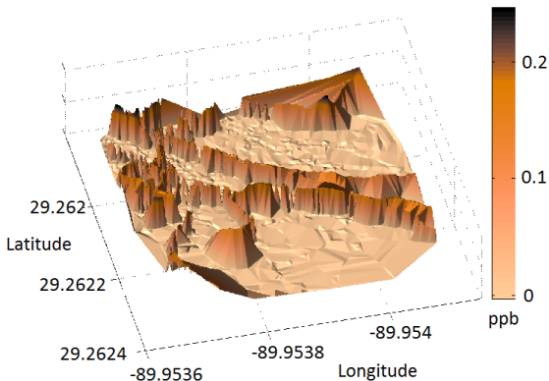


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# Our Other Adaptive Control Applications

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Joint work with J. Muse from AFRL on model reference adaptive control to reduce oscillations, applied to hovering helicopters.

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**Thanks for your interest!**