Adaptive Tracking and Parameter Identification

Michael Malisoff
Basic Problem Formulation

Consider a system of differential equations

\[ \dot{\xi} = f(\xi, P, u) \]  \hspace{1cm} (1)

with a vector \( P \) of unknown constant parameters and functions \( \xi_R \) and \( u_R \) such that \( \dot{\xi}_R(t) = f(\xi_R(t), P, u_R(t)) \) for all \( t \geq 0 \).
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Problem: Find \( u(\xi, \hat{P}) \) and a system of differential equations

\[ \dot{\hat{P}} = g(\xi, \hat{P}) \]  \hspace{1cm} (2)

such that with the control choice \( u(\xi, \hat{P}) \) in (1), all solutions \( Y = (\tilde{\xi}, \tilde{P}) = (\xi - \xi_R, P - \hat{P}) \) converge to 0 as \( t \to +\infty \).
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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...
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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...
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Flight control, mechanical systems, robotics,...
Adaptive Robotic Curve Tracking
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Our more general settings: Perturbations and state constraints, motivated by our robotics field work at Grand Isle, Louisiana.
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Gyroscopic models: Steering command control for convergence to parallel motion to, but positive distance from, curve.
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ZP. Jiang, E. Justh, P. Krishnaprasad, V. Lumelsky, A. Stepanov
We use Lyapunov functions for systems of the form \( \dot{Y} = G(Y) \).

Nonstrict (resp., strict) Lyapunov functions are continuously differentiable proper positive definite \( V \)'s that satisfy the nonstrict (resp., strict) decay condition along all solutions of the systems.

Positive definiteness:
\[ V(E) = 0, \quad V(Y) > 0 \quad \text{for all} \quad Y \neq E. \]

Properness:
\[ V(Y) \to +\infty \quad \text{as} \quad |Y| \to +\infty \quad \text{or as} \quad Y \text{ converges to the boundary of the state space while staying in the state space.} \]

Nonstrict decay:
\[ \frac{d}{dt} V(Y(t)) \leq 0 \quad \text{along all solutions of system.} \]

Strict decay: there is a continuous positive definite \( \alpha \) such that
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Gyroscopic Model (with Georgia Tech)

\[ \rho = |r_2 - r_1|, \quad \phi = \text{angle between } x_1 \text{ and } x_2, \quad \cos(\phi) = \frac{x_1 \cdot x_2}{4/10} \]
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Simpler 2D case: Boundary following with gyroscopic control.

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Curve Tracking Dynamics for 2D

\[ \dot{\rho} = -\sin(\phi) \]
\[ \dot{\phi} = \kappa \cos(\phi) \]
\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho), \text{equilibrium } E = (\rho_0, 0) \]

Along all solutions of (CL) for all \( t \geq 0 \), we have \( dV(\rho, \phi) \leq 0 \).
Curve Tracking Dynamics for 2D

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\begin{cases}
\dot{\rho} = -\sin(\phi) \\
\dot{\phi} = \frac{\kappa \cos(\phi)}{1 + \kappa \rho} - u_b, \quad (\rho, \phi) \in \mathcal{X} = (0, +\infty) \times (-\pi/2, \pi/2)
\end{cases}
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\[ u_b = \frac{\kappa \cos(\phi)}{1 + \kappa \rho} - h'(\rho) \cos(\phi) + \mu \sin(\phi) \tag{4} \]
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h(\rho) = \alpha \left\{ \rho + \frac{\rho_0^2}{\rho} - 2\rho_0 \right\}, \quad \rho_0 = \text{desired value for } \rho
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\[\dot{\rho} = -\sin \phi, \quad \dot{\phi} = h'(\rho) \cos \phi - \mu \sin \phi \tag{CL}\]

\[V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho), \quad \text{equilibrium } \mathcal{E} = (\rho_0, 0) \tag{6}\]

Along all solutions of (CL) for all \( t \geq 0 \), we have \( \frac{d}{dt} V(\rho, \phi) \leq 0 \).
Strict Lyapunov Function (Mazenc-M-Z, TAC)

**Theorem 1:** The closed loop system (CL) has the strict Lyapunov function

\[ U(Y) = -h'(\rho) \sin(\phi) + \frac{1}{\mu} \int_0^{V(\rho,\phi)} \gamma(m) dm + \Gamma(V(\rho,\phi)) + V(\rho,\phi), \]

where \( \gamma(q) = \frac{2(q+2\rho_0)^3}{\rho_0^4} + 1 + 0.5\mu^2 + \mu, \) \( Y = (\rho - \rho_0, \phi), \)

\[ \Gamma(q) = \frac{18}{\rho_0} q + 9 \left( \frac{2}{\rho_0} \right)^4 q^4, \text{ and } V(\rho,\phi) = -\ln \left( \frac{\cos(\phi)}{\rho_0} \right) + h(\rho) \]

on its state space \( \mathcal{X} = (0, +\infty) \times (-\pi/2, \pi/2). \) \( \blacksquare \)
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on its state space \( \mathcal{X} = (0, +\infty) \times (-\pi/2, \pi/2). \)

\[ U(Y(t)) \geq V(\rho(t), \phi(t)) \quad \text{(PD)} \]

\[ \frac{d}{dt} U(Y(t)) \leq -0.5[h'(\rho(t)) \cos(\phi(t))]^2 - \sin^2(\phi(t)) \quad \text{(SD)} \]
Unknown Control Gains (M-Zhang)

\[
\begin{align*}
\dot{\rho} &= -\sin(\phi) \\
\dot{\phi} &= \frac{\kappa \cos(\phi)}{1+\kappa \rho} + Ku, \quad K \in (c_{\text{min}}, c_{\text{max}}) \subseteq (0, \infty) \\
\dot{\hat{K}} &= (\hat{K} - c_{\text{min}})(c_{\text{max}} - \hat{K}) \frac{\partial U}{\partial \phi} u, \quad \hat{K} \in (c_{\text{min}}, c_{\text{max}})
\end{align*}
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\begin{aligned}
\dot{\rho} & = -\sin(\phi) \\
\dot{\phi} & = \frac{\kappa \cos(\phi)}{1+\kappa \rho} + K u, \quad K \in (c_{\text{min}}, c_{\text{max}}) \subseteq (0, \infty) \\
\dot{\hat{K}} & = (\hat{K} - c_{\text{min}})(c_{\text{max}} - \hat{K}) \frac{\partial u}{\partial \phi} u, \quad \hat{K} \in (c_{\text{min}}, c_{\text{max}})
\end{aligned}
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\[u(\rho, \phi, \hat{K}) = -u_b(\rho, \phi) / \hat{K}.
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Built strict Lyapunov functions for

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\begin{align*}
\dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\
\dot{\tilde{q}}_2 &= \frac{\kappa \cos(\tilde{q}_2)}{1 + \kappa (\tilde{q}_1 + \rho_0)} - \frac{K}{\hat{K} + K} u_b \\
\dot{\tilde{K}} &= -(\tilde{K} + K - c_{\min})(c_{\max} - \tilde{K} - K) \frac{\partial U}{\partial \phi} \frac{u_b}{\hat{K} + K}
\end{align*}
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i.e., the dynamics for \( Y = (\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K) \).
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i.e., the dynamics for \(Y = (\tilde{q}_1, \tilde{q}_2, \hat{K}) = (\rho - \rho_0, \phi, \hat{K} - K)\).
\(\xi_R = (\rho_0, 0)\). Strictness allowed a robustness analysis to satisfy performance and safety bounds under other uncertainties.
Field Work at Grand Isle, LA

20 days of field work off Grand Isle. Search for oil spill remnants. Georgia Tech Savannah Robotics (co-led by Fumin Zhang)
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Hyperlinked Related References

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Our Other Adaptive Control Applications

- Brushless DC motors turning a mechanical load with uncertain motor electric parameters including integral ISS analysis.
- Variants for uncertain parameters that enter the system in a nonlinear way for curve tracking with unknown curvatures.
- To also allow delays in state observations in our controls, we convert our strict LF into Lyapunov-Krasovskii functionals.
- We used artificial neural network expansions for extensions to cases where the parameter need not be constant.
- Joint work with J. Muse from AFRL on model reference adaptive control to reduce oscillations, applied to hovering helicopters.
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Thanks for your interest!