Delay Compensation in Control Systems

Michael Malisoff

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M. Jankovic, M. Krstic, Z. Lin, SI. Niculescu, P. Pepe, A. Teel,...

Stability Definitions

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$$x'(t) = \mathcal{G}(t, x(t), x(t-\tau)), \quad x(t) \in \mathcal{X} \tag{Σ}$$

$$|x(t)| \le \gamma_1 \left(e^{t_0 - t}\gamma_2(|x|_{[t_0-\tau, t_0]})\right) \tag{UGAS}$$

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$$x'(t) = \mathcal{G}\big(t, x(t), x(t-\tau), \delta(t)\big), \quad x(t) \in \mathcal{X} \qquad (\Sigma_{\text{pert}})$$

$$|x(t)| \le \gamma_1 \left(e^{t_0 - t} \gamma_2(|x|_{[t_0-\tau, t_0]}) \right) + \gamma_3(|\delta|_{[t_0, t]}) \qquad \text{(ISS)}$$

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Find γ_i's by building Lyapunov-Krasovskii functionals (LKFs).

Definition in Control-Affine Case

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t))[u_s(t, x(t - \tau)) + \delta(t)]. \qquad (\Sigma_{\mathrm{d}})$$

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Assume: f and g are locally Lipschitz and grow linearly in x, $u_s \in C^1$, $|(\partial u_s/\partial x)(t, x)|$ bounded. $x_t(s) = x(t + s)$, $-\tau \le s \le 0$.

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Definition: A function $U : [0, \infty) \times \mathcal{C}_n([-\tau, 0]) \to [0, \infty)$ is an ISS-LKF for (Σ_d) provided there are $\alpha_i \in \mathcal{K}_\infty$ such that for all solutions $x(t)$ of (Σ_d), $U(t, x_t)$ is absolutely continuous in t and

(i) $\alpha_1(|\phi(0)|) \le U(t, \phi) \le \alpha_2(|\phi|_{[-\tau,0]})$ and

(ii) $\frac{d}{dt}\big(U(t, x_t)\big) \le -\alpha_3(U(t, x_t)) + \alpha_4(|\delta|_{[t_o,t]})$

hold for all $\phi \in \mathcal{C}_n([-\tau, 0])$ and almost all $t \ge t_o$ and all $t_0 \ge 0$.

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Malisoff, M., and F. Zhang, "Robustness of adaptive control under time delays for three-dimensional curve tracking," *SIAM Journal on Control and Optimization*, 53(4):2203-2236, 2015.

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Assumption L: There are $\sigma \in \mathcal{K}_\infty$ such that $\sigma(r) \leq r$ for all $r \geq 0$; constants $K_1 \geq 1$ and $K_i \geq 0$ for $i = 2, 3, 4$; and a C^1 uniformly proper and positive definite $V : [0, \infty) \times \mathbb{R}^n \to [0, \infty)$ such that for all $x \in \mathbb{R}^n$, $q \in \mathbb{R}^n$, $l \geq 0$, and $t \geq 0$, we have

- H1 $V_t(t, x) + V_x(t, x)[f(t, x) + g(t, x)u_s(t, x)] \leq -\sigma^2(|x|)$;
- H2 $|V_x(t, x)g(t, x)| \leq K_1\sigma(|x|)$, $\left|\frac{\partial u_s}{\partial x}(t, x)f(l, x)\right|^2 \leq K_2\sigma(|x|)^2$;
- H3 $\left|\frac{\partial u_s}{\partial x}(t, x)g(l, x)\right|^2 \leq K_3(\sigma(|x|) + 1)$; and
- H4 $\left[\left|\frac{\partial u_s}{\partial x}(t, x)g(l, x)\right| |u_s(l, q)|\right]^2 \leq K_4[\sigma^2(|x|) + \sigma^2(|q|)]$.

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H4 $\left[\left|\frac{\partial u_s}{\partial x}(t, x)g(l, x)\right| |u_s(l, q)|\right]^2 \leq K_4[\sigma^2(|x|) + \sigma^2(|q|)]$.

Exponentially stable $\dot{x}(t) = (A(t) + B(t)K(t))x(t)$ with $\sigma(s) = s$

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Thm 1: (M-Mazenc-Lin, '08) If Assumption L holds, then

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t))[u_s(t, x(t - \tau)) + \delta(t)] \qquad (\Sigma_{\mathrm{d}})$$

with any constant feedback delay $\tau \in (0, \bar{\tau}]$ where

$$\bar{\tau} = \frac{1}{4K_1\sqrt{3K_2+3K_4+1}}$$

admits the ISS-LKF

$$U(t, x_t) = V(t, x(t)) + \frac{1}{8\bar{\tau}}\int_{t-2\bar{\tau}}^{t}\left(\int_r^t \sigma^2(|x(p)|)\mathrm{d}p\right)\mathrm{d}r$$

and therefore is ISS. □

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Delay need not be known. Can drop delay bound in many cases using reduction or prediction or scaling of controls.

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$$\dot{x}(t) = M(t)x(t) + N(t)u(t-\tau) + \delta(t). \tag{1}$$

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Thm 2: (MMN'14) If there is a bounded continuous K such that

$$\dot{z}(t) = \big[M(t) + \lambda(t, t+\tau)N(t+\tau)K(t)\big]z(t) \qquad (2)$$

is UGAS, where λ is the fundamental matrix for M, then there are functions $\gamma_i \in \mathcal{K}_\infty$ such that all trajectories of (1) with

$$u(t) = K(t)\left[x(t) + \int_{t-\tau}^{t} \lambda(t, r+\tau)N(r+\tau)u(r)\mathrm{d}r\right] \qquad (3)$$

satisfy

$$|x(t)| + |u|_{[t-\tau,t]} \leq \gamma_1\Big(\gamma_2\big(|x(t_0)| + |u|_{[t_0-\tau,t_0]}\big)e^{t_0-t}\Big) + \gamma_3\big(|\delta|_{[t_0,t]}\big)$$

for all initial times $t_0 \geq 0$ and all $t \geq t_0$.

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Assumption 1: The functions A and B are bounded and continuous, and there is a known bounded continuous function $K : [0, \infty) \to \mathbb{R}^{\ell \times n}$ such that $\dot{x}(t) = [A(t) + B(t)K(t)]x(t)$ is uniformly globally exponentially stable to 0.

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Assumption 2: The function $h : \mathbb{R} \to [0, \infty)$ is C^1 and bounded from above by a constant $c_h > 0$. Also, its derivative $\dot{h}$ is bounded from below, and $\dot{h}$ is bounded from above by a constant $l_h \in (0, 1)$, and $\dot{h}$ has a global Lipschitz constant $n_h > 0$.

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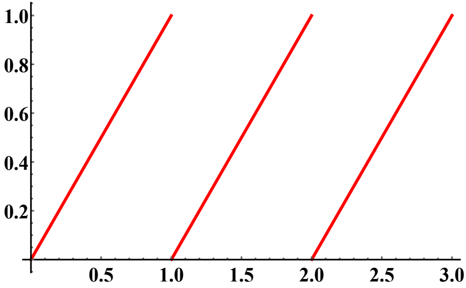

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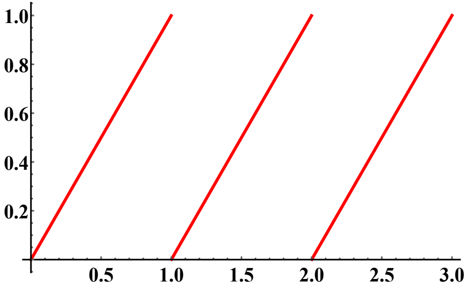

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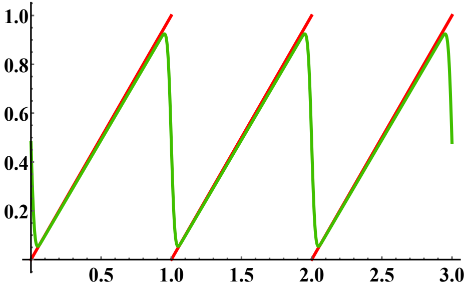

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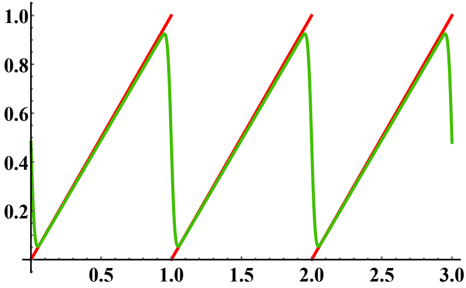

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Assumption 2 holds with $c_h = 0.924$, $l_h = 0.98$, and $n_h = 592.72$.

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We use an $p n$-dimensional dynamic extension to build our delay compensating control for any number of predictors

$$p > \max\left\{2, 4\left(\frac{b_1}{\sqrt{2}} + b_2\right)\frac{c_h}{1-l_h}\right\}, \tag{LB}$$

where

$$b_1 = \left[1 + \left(1 + \frac{u_c}{p}\right)^p |A|_\infty\right]\left(1 + \frac{u_c}{p}\right)^p |A|_\infty,$$

$$b_2 = \left[1 + \left(1 + \frac{u_c}{p}\right)^p |A|_\infty\right]^2\left(1 + \frac{u_c}{p}\right), \text{ and } u_c = \frac{c_h n_h}{(1-l_h)^2} + \frac{l_h}{1-l_h}.$$

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$\Omega_i(t) = t - (i/p)h(t)$ and $\theta_i(t) = \Omega^{-1}_{p-i+1}(\Omega_{p-i}(t))$ for $i \in \{0, ..., p\}$

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p sequential predictors for $\dot{x}(t) = A(t)x(t) + B(t)u(t - h(t))$

$\Omega_i(t) = t - (i/p)h(t)$ and $\theta_i(t) = \Omega_{p-i+1}^{-1}(\Omega_{p-i}(t))$ for $i \in \{0, ..., p\}$

$R_1(t) = \dot{\theta}_1(t)$, $R_i(t) = \dot{\theta}_i(t)R_{i-1}(\theta_i(t))$ for $i > 1$.

Third Approach: Sequential Predictors

Thm 3: (M-Mazenc, '17) Let Assumptions 1-2 hold and p satisfy (LB). Then if we use the control $u(t) = K(\Omega_p^{-1}(t))z_p(t)$ in (LTV), where z_p is the last n components of the system

$$\begin{aligned} \dot{z}_1(t) &= R_1(t)A(\theta_1(t))z_1(t) + R_1(t)B(\theta_1(t))u(\Omega_{p-1}(t)) \\ &\quad + L_1(t)[z_1(\theta_1^{-1}(t)) - x(t)] \\ \dot{z}_i(t) &= R_i(t)A(G_i(t))z_i(t) + R_i(t)B(G_i(t))u(\Omega_{p-i}(t)) \\ &\quad + L_i(t)[z_i(\theta_i^{-1}(t)) - z_{i-1}(t)],\ i \in \{2, \dots, p\} \end{aligned} \tag{4}$$

where $L_i(t) = -I_n - R_i(t)A(G_i(t))$ and $G_i = \Omega_p^{-1} \circ \Omega_{p-i}$, then the dynamics for $(x, \mathcal{E})$ are globally exponentially stable to 0, where $\mathcal{E}(t) = (z_1(t) - x(\theta_1(t)), z_2(t) - z_1(\theta_2(t)), \dots, z_p(t) - z_{p-1}(\theta_p(t)))$.

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Thank you for your attention!