Systems and Control: An Introduction and a Marine Robotics Application

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What Do We Mean By Control Systems?

These are triply parameterized families of ODEs of the form

\[ Y'(t) = F(t, Y(t), u(t, Y(t - \tau)), \Gamma, \delta(t)) \],

\[ Y(t) \in Y \].

(1)

\[ Y \subseteq \mathbb{R}^n \].

\[ \delta: [0, \infty) \to D \] represents uncertainty.

\[ D \subseteq \mathbb{R}^m \].

The vector \( \Gamma \) is constant but unknown.

\( \tau \) is a constant delay.

Specify \( u \) to get a doubly parameterized closed loop family

\[ Y'(t) = G(t, Y(t), Y(t - \tau), \Gamma, \delta) \],

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(2)

where \( G(t, Y(t), Y(t - \tau), \Gamma, \delta) = F(t, Y(t), u(t, Y(t - \tau)), \Gamma, \delta) \).

Problem: Given a desired reference trajectory \( Y_r \), specify \( u \) and an estimate \( \hat{\Gamma} \) of \( \Gamma \) such that the augmented error \( E(t) = (Y(t) - Y_r(t), \Gamma - \hat{\Gamma}(t)) \) satisfies ISS with respect to \( \delta \).
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ISS (Sontag, ’89) generalizes global asymptotic stability.

\[ Y'(t) = G(t, Y(t), Y(t-\tau), \Gamma), \quad Y(t) \in Y(\Sigma) \]

\[ |Y(t)| \leq \gamma_1(e^{t_0-t} - t \gamma_2(|Y|_{[t_0-\tau, t_0]}) \] (UGAS)

Our \( \gamma_i \)'s are 0 at 0, strictly increasing, and unbounded. \( \gamma_i \in K_\infty \).

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For many systems, we design controls $u$ that ensure ISS under the delays $\tau$ and uncertainties $\delta$ that prevail in engineering.
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We combine the plants with dynamics for parameter estimators $\hat{\Gamma}(t)$ that converge to $\Gamma$, and then use $\hat{\Gamma}(t)$ in $u$, instead of $\Gamma$. 
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2D Curve Tracking for Marine Robots

\[ \rho = |r_2 - r_1|, \quad \phi = \text{angle between } x_1 \text{ and } x_2, \quad \cos(\phi) = \frac{x_1 \cdot x_2}{\|x_1\| \|x_2\|} \]
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Two-Dimensional Curve Tracking Model

\[ \dot{\rho} = -\sin \phi, \quad \dot{\phi} = \kappa \cos \phi + \kappa \rho - u, \]

\[ X = (\rho, \phi) \in \Omega. \]

\[ \rho = \text{relative distance}, \quad \phi = \text{bearing}, \quad X = (0, +\infty) \times (-\pi/2, \pi/2). \]

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Control Objectives in Undelayed Nonadaptive Case:

(A) Construct \( u \) to get UGAS of an equilibrium \( X_0 = (\rho_0, 0) \).

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ISS:

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Why Can’t We Apply Feedback Linearization?

In the new variables $\rho$ and $z = \sin(\phi)$, the system \(\Sigma\) becomes

\[
\dot{\rho} = -z, \\
\dot{z} = \kappa (1 - z^2)^{1/2} + \kappa \rho - u \sqrt{1 - z^2} \quad (\Sigma_c)
\]

on the new state space \(X_c = (0, \infty) \times (-1, 1)\).

The control \(u_{fl} = \frac{1}{\sqrt{1 - z^2}} (\kappa (1 - z^2)^{1/2} + \kappa \rho - K_1 (\rho - \rho_0)) + K_2 z)\) for any constants \(K_i > 0\) gives the closed loop dynamics

\[
\dot{\rho} = -z, \\
\dot{z} = K_1 (\rho - \rho_0) - K_2 z. \\
(3)
\]

Proposition: There do not exist constants \(K_1 > 0\) and \(K_2 > 0\) such that \(X_c = (0, \infty) \times (-1, 1)\) is forward invariant for (3).
Why Can’t We Apply Feedback Linearization?

In the new variables $\rho$ and $z = \sin(\phi)$, the system $(\Sigma)$ becomes

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\end{align*}$$

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They realized the nonadaptive UGAS objective using
\[ u = \kappa \cos(\phi) + \kappa \rho - h'(\rho) \cos(\phi) + \mu \sin(\phi). \]

Assumption 1:
\[ h : (0, +\infty) \rightarrow [0, \infty) \] is \( C^1 \), \( h' \) has only finitely many zeros, \( \lim_{\rho \to 0^+} h(\rho) = \lim_{\rho \to \infty} h(\rho) = \infty \), and \( h \in \text{PD}(\rho_0) \).

Strategy:
Use the Lyapunov function candidate
\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho). \]

Along
\[ \dot{\rho} = -\sin(\phi), \quad \dot{\phi} = h'(\rho) \cos(\phi) - \mu \sin(\phi), \]
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Review of Zhang-Justh-Krishnaprasad CDC’04

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Extra Properties to Achieve All Of Our Goals

To realize our goals, we added assumptions on which hold for

\[ h(\rho) = \alpha(\rho + \rho_o/\rho - 2\rho_o) \]

See my Automatica and TAC papers with Fumin Zhang.
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See my Automatica and TAC papers with Fumin Zhang.
Our Adaptive Robust Curve Tracking Controller

\[ \dot{\rho} = -\sin(\phi) \]

\[ \dot{\phi} = \kappa \cos(\phi) \]

\[ 1 + \kappa \rho \left( \Gamma[\hat{u} + \delta] \right) \]

\[ \rho, \phi \in \text{full state space} \]

\[ u(\rho, \phi, \hat{\Gamma}) = -\frac{1}{\hat{\Gamma}} \left( \kappa \cos(\phi) \right) \]

\[ \hat{\Gamma} = (\hat{\Gamma} - c_{\text{min}}) \left( c_{\text{max}} - \hat{\Gamma} \right) \]

\[ V^\#(\rho, \phi) = -h'(\rho) \sin(\phi) + \int_0^V(\rho, \phi) \gamma(m) \, dm \]

\[ \gamma(q) = \frac{1}{\mu} \left( \frac{2}{\alpha^2 \rho^4} \right)^{\frac{3}{2}} + \frac{1}{\mu^2} + \frac{2}{\alpha^2 \rho^4} + \frac{576}{\alpha^2 q^3} \]

\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho) \]
Our Adaptive Robust Curve Tracking Controller

\[
\begin{align*}
\dot{\rho} &= -\sin(\phi) \\
\dot{\phi} &= \frac{\kappa \cos(\phi)}{1 + \kappa \rho} + \Gamma[u + \delta]
\end{align*}
\]

\( (\rho, \phi) \in (0, \infty) \times (-\pi/2, \pi/2) \) (\( \Sigma_c \))
Our Adaptive Robust Curve Tracking Controller

\[
\begin{cases}
\dot{\rho} = -\sin(\phi) \\
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\end{cases}
\]

full state space \( (\rho, \phi) \in (0, \infty) \times (-\pi/2, \pi/2) \) \((\Sigma_c)\)

Control: \( u(\rho, \phi, \hat{\Gamma}) = -\frac{1}{\hat{\Gamma}} \left( \frac{\kappa \cos(\phi)}{1 + \kappa \rho} - h'(\rho) \cos(\phi) + \mu \sin(\phi) \right) \) \((7)\)

Estimator: \( \dot{\hat{\Gamma}} = (\hat{\Gamma} - c_{\min})(c_{\max} - \hat{\Gamma}) \frac{\partial V^\sharp(\rho, \phi)}{\partial \phi} u(\rho, \phi, \hat{\Gamma}) \) \((8)\)
Our Adaptive Robust Curve Tracking Controller

\[
\left\{ \begin{array}{l}
\dot{\rho} = -\sin(\phi) \\
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\end{array} \right. \quad (\rho, \phi) \in \mathcal{S}_c \quad \text{full state space}
\]

Control: \quad u(\rho, \phi, \hat{\Gamma}) = -\frac{1}{\hat{\Gamma}} \left( \frac{\kappa \cos(\phi)}{1+\kappa \rho} - h'(\rho) \cos(\phi) + \mu \sin(\phi) \right) \quad (7)

Estimator: \quad \dot{\hat{\Gamma}} = (\hat{\Gamma} - \hat{\kappa}_{\text{min}})(\hat{\kappa}_{\text{max}} - \hat{\Gamma}) \frac{\partial V^\#(\rho, \phi)}{\partial \phi} u(\rho, \phi, \hat{\Gamma}) \quad (8)

\[ V^\#(\rho, \phi) = -h'(\rho) \sin(\phi) + \int_0^{V(\rho, \phi)} \gamma(m) \, dm \quad (9) \]

\[ \gamma(q) = \frac{1}{\mu} \left( -\frac{2}{\alpha^2 \rho_0^4} (q + 2\alpha \rho_0)^3 + 1 \right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4 \alpha^2} q^3 \quad (10) \]

\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho) \quad (11) \]
Robustly Forwardly Invariant Hexagonal Regions

Restrict the perturbations $\delta(t)$ to keep the state $X = (\rho, \phi)$ from leaving the state space $X = (0, \infty) \times (-\pi/2, \pi/2)$.

View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a nested union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq ... \subseteq H_i \subseteq ...$.

For each $i$, all trajectories of $(\Sigma_c)$ starting in $H_i$ for all $\delta : [0, \infty) \rightarrow [-\delta^*_i, \delta^*_i]$ stay in $H_i$.

The tilted legs have slope $c_{\min} \mu/c_{\max}$.

For each index $i$, we take $\delta^*_i$ to be the largest allowable disturbance bound to maintain forward invariance of $H_i$.

Then we prove ISS of the tracking and parameter identification system on each set $H_i$, with the disturbance set $D = [-\delta^*_i, \delta^*_i]$. 
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Field Work at Grand Isle, LA

20 days of field work off Grand Isle.

Search for oil spill remnants.

Georgia Tech Savannah Robotics Team (led by Fumin Zhang).
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Circle Tracking by ASV Victoria
Line Tracking by ASV Victoria
Crude Oil Concentration Maps
Conclusions

Adaptive nonlinear controllers are useful for many engineering control systems with delays and uncertainties. Curve tracking controllers for autonomous marine vehicles are important for monitoring water quality, especially after oil spills. Our controls identify parameters and are adaptive and robust to the perturbations and delays that arise in field work. We can prove these properties using input-to-state stability, dynamic extensions, and Lyapunov-Krasovskii functionals. We used our controls on student built marine robots to map residual crude oil from the Deepwater Horizon spill. In our future work, we will study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.
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References for 2D Case with Hyperlinks


