Systems and Control: An Introduction and a Marine Robotics Application

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Problem: Given a desired reference trajectory Y_r, specify u and a dynamics for an estimate $\hat{\Gamma}$ of Γ such that the augmented error $\mathcal{E}(t) = (Y(t) - Y_r(t), \Gamma - \hat{\Gamma}(t))$ satisfies ISS with respect to δ.

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When $\tau = 0$, a system is ISS iff it has an ISS LF (Sontag-Wang).

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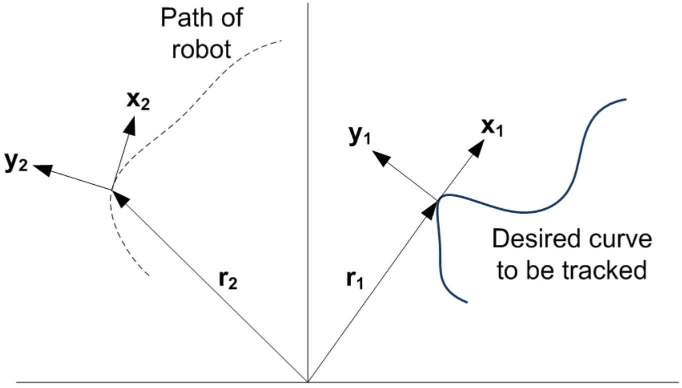

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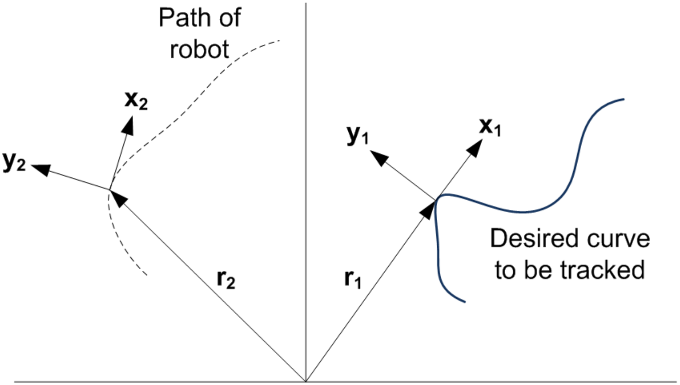

$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

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Feedback linearization with $z = \sin(\phi)$ cannot be applied.

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In the new variables ρ and $z = \sin(\phi)$, the system (Σ) becomes

$$\dot{\rho} = -z, \quad \dot{z} = \frac{\kappa(1-z^2)}{1+\kappa\rho} - u\sqrt{1-z^2} \qquad (\Sigma_c)$$

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Proposition: There do not exist constants $K_1 > 0$ and $K_2 > 0$ such that $\mathcal{X}_c = (0, \infty) \times (-1, 1)$ is forward invariant for (3).

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This gives UGAS, using LaSalle Invariance.

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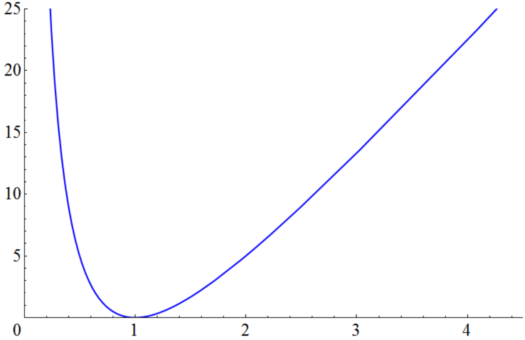

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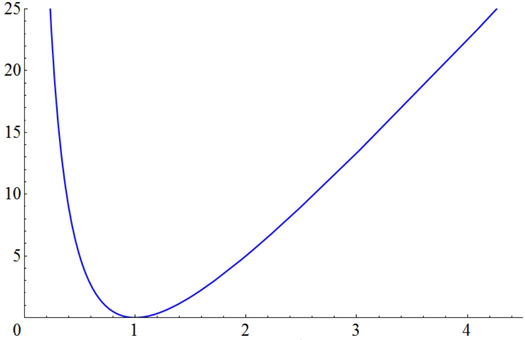

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See my Automatica and TAC papers with Fumin Zhang.

Our Adaptive Robust Curve Tracking Controller

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$$\begin{cases} \dot{\rho} = -\sin(\phi) \\ \dot{\phi} = \frac{\kappa\cos(\phi)}{1+\kappa\rho} + \Gamma[u + \delta] \end{cases} \quad (\rho, \phi) \in \overbrace{(0, \infty) \times (-\pi/2, \pi/2)}^{\text{full state space}} \quad (\Sigma_c)$$

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$$V^{\sharp}(\rho,\phi) = -h'(\rho)\sin(\phi) + \int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m \qquad (9)$$

$$\gamma(q) = \frac{1}{\mu}\left(\frac{2}{\alpha^2\rho_0^4}(q+2\alpha\rho_0)^3 + 1\right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4\alpha^2}q^3 \qquad (10)$$

$$V(\rho,\phi) = -\ln\left(\cos(\phi)\right) + h(\rho) \qquad (11)$$

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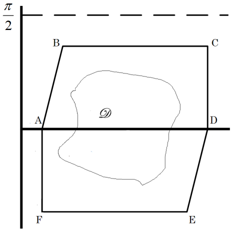

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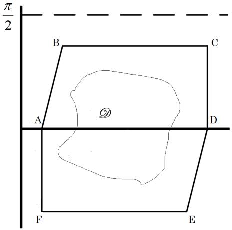

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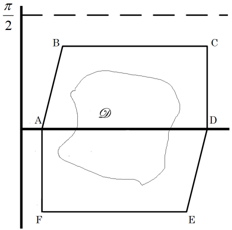

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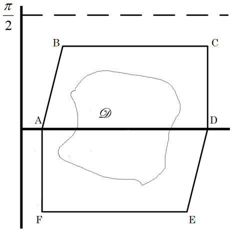

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For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Then we prove ISS of the tracking and parameter identification system on each set H_i, with the disturbance set $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

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Georgia Tech Savannah Robotics Team (led by Fumin Zhang).

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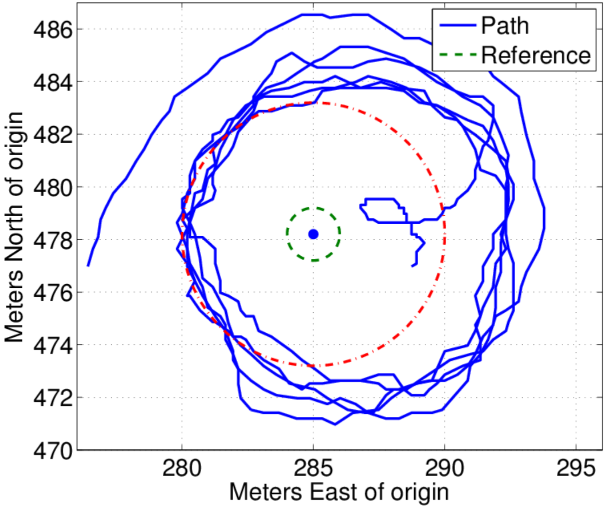
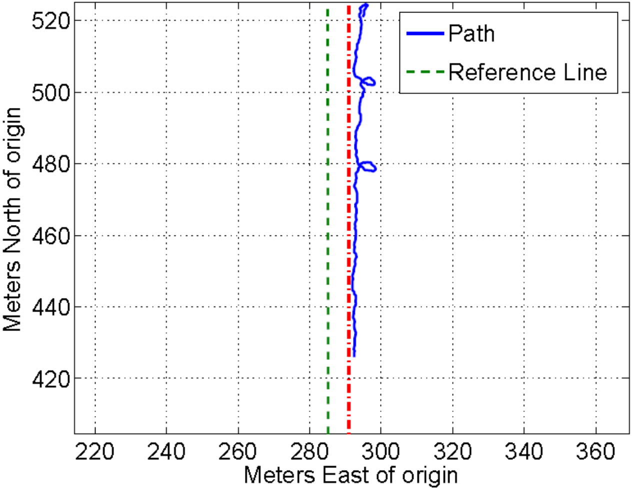
(Loading Video...)

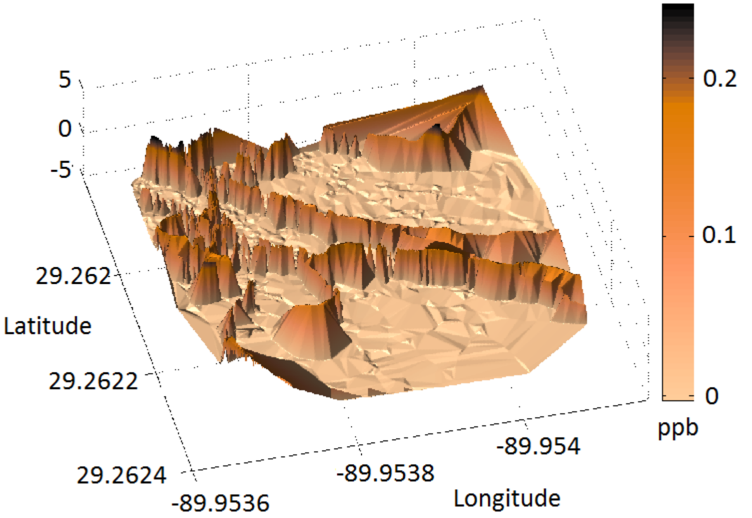
Circle Tracking by ASV Victoria



Line Tracking by ASV Victoria



Crude Oil Concentration Maps



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In our future work, we will study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.

References for 2D Case with Hyperlinks

Malisoff, M., F. Mazenc, and F. Zhang, “Stability and robustness analysis for curve tracking control using input-to-state stability," *IEEE Transactions on Automatic Control*, Volume 57, Issue 5, May 2012, pp. 1320-1326.

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References for 3D Case with Hyperlinks

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