Adaptive Tracking and Parameter Estimation with Unknown High-Frequency Control Gains: A Case Study in Strictification

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Find γ_i's by building certain strict LFs for $\dot{Y} = \mathcal{G}(t, Y, 0)$.

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- Main PE Assumption: positive definiteness of the matrices

$$\mathcal{P}_i \stackrel{\text{def}}{=} \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \in \mathbb{R}^{(p_i+1)\times(p_i+1)},\ 1 \leq i \leq s \tag{4}$$

where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for each i.

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The estimator and feedback can only depend on things we know.

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Tracking error: $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R)$
Parameter estimation errors: $\widetilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\widetilde{\psi}_i = \psi_i - \hat{\psi}_i$
Estimators: $\hat{\theta}_i = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_i})$ and $\hat{\psi} = (\hat{\psi}_1, \ldots, \hat{\psi}_s)$

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$$\begin{aligned} \mathcal{X} &= \mathbb{R}^{r+s} \times \left(\prod_{i=1}^{s} \left\{ \prod_{j=1}^{p_i} (\theta_{i,j} - \theta_M, \theta_{i,j} + \theta_M) \right\} \right) \\ & \quad \times \left(\prod_{i=1}^{s} \left(\psi_i - \overline{\psi}, \psi_i - \underline{\psi} \right) \right) \subseteq \mathbb{R}^{r+s+p_1+\ldots p_s+s} \,. \end{aligned}$$

Stabilization Analysis

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- We build a global strict LF for the augmented error $Y = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \in \mathcal{X}$ dynamics.

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Transformation from Our Paper

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Theorem: We can construct $K \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \doteq K\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Upsilon}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \ , \qquad (11)$$

$$\begin{aligned} \text{where} \quad \overline{\Upsilon}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) &= -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \\ &\quad + \tfrac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \ , \end{aligned} \qquad (12)$$

$$\lambda_i(t) = \big(k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t))\big) \ , \qquad (13)$$

$$\begin{aligned} \alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) &= \begin{bmatrix} \tilde{\theta}_i\psi_i - \theta_i\tilde{\psi}_i \\ \widetilde{\psi}_i \end{bmatrix} \ , \quad \text{and} \\ \Omega_i(t) &= \textstyle\int_{t-T}^{t}\int_{m}^{t}\lambda_i^\top(s)\lambda_i(s)\mathrm{d}s\,\mathrm{d}m \ , \end{aligned} \qquad (14)$$

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- It would be useful to extend to cover models that are not affine in Γ, feedback delays, and output feedbacks.