Tracking Control and Robustness for Planar Vertical Takeoff and Landing Aircraft under Bounded Feedbacks

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1. PVTOL Model (Hauser-Sastry-Meyer, 1992)

This benchmark system models aircraft position and roll angle.

\[ \begin{align*}
\dot{x} &= v_1 \cos(\theta) + \epsilon v_2 \cos(\theta) \\
\dot{y} &= v_2 \cos(\theta) + \epsilon v_1 \sin(\theta) \\
\dot{\theta} &= a - \delta
\end{align*} \]  

where \(a\) is the thrust control directed out of the bottom, \(\epsilon\) is the rolling moment control, \(\dot{x}\) is the coupling between the roll moment and lateral acceleration.

2. Change of Coordinates (Olfati-Saberi, 2002)

The Olfati-Saberi change of variables \( z_1 = x - \epsilon \sin(\theta), z_2 = y + \epsilon \cos(\theta) - 1, w_1 = y - \epsilon \sin(\theta), w_2 = \epsilon \cos(\theta), \) and \(\xi = 0\) give

\[ \begin{align*}
\dot{z}_1 &= v_1 - v_2 \sin(\xi) \\
\dot{z}_2 &= v_2 \cos(\xi) \\
w_1 &= y - \epsilon \sin(\theta) \\
w_2 &= \epsilon \cos(\theta) \\
\dot{\xi} &= \epsilon
\end{align*} \]

The new system is \( Q = (z_1, z_2, w_1, w_2, \xi)^T \).

3. Tracking Dynamics and Objective

Choose any \( C^2 \) reference trajectory-input pair for \(Q\) such that

a) \(z_1(\cdot), v_1(\cdot), v_2(\cdot) \) are \(C^2\) functions such that \(z_1(\cdot) \in [-\epsilon^2 + \epsilon \sqrt{2}, \epsilon^2 - \epsilon \sqrt{2}], v_1(\cdot) \geq 0, v_2(\cdot) \geq 0,\)

b) \(z_2\) and \(w_1\) are bounded, and

c) \(w_1 = \langle u_w, n_1 \rangle^T, n_1, w_2\) are bounded and \(\langle u_w, n_2 \rangle = 0.\)

5. Bounded Thrust (BT) Controller

We can also track along Cassini’s Oval.

6. Transformed Tracking Dynamics (TTD)

We can choose constants \(a > 0\) and \(\lambda > 0\) such that

\[ \frac{\dot{z}_1}{\lambda} = \frac{z_1 - z_1^*}{\lambda} - \frac{z_2}{\lambda} - \frac{v_1(\cdot)}{\lambda} - \frac{v_2(\cdot)}{\lambda}\]

is bounded, \(C^1\), and renders (TTD) both UGAS and ULES to 0.

7. Designing \( u_w \)

We anticipate being useful for other models in feedforward form.

8. Input-to-State Stability (ISS)

For any \( \delta > 0, \) we can scale \(z_1\) and \(v_1\) to prove ISS for (TTD) under disturbances \( \delta (z_1, z_2, v_1, v_2) \) added to \(w_1, w_2\).

We developed a new bounded tracking feedback design that gives UGAS and ULES for a large class of reference trajectories.

10. Trackable Trajectories

If \(z_1(\cdot), w_1(\cdot) : [0, \infty) \to \mathbb{R}^2\) is any \(C^2\) time-periodic function such that \(w_2(\cdot) > 0\), then we can track using

\[ u_w = \sqrt{v_1(\cdot) + v_2(\cdot) + \delta} \]

and with \(z_2 = 0, z_1 = 0, v_1 = 0, v_2 = 0,\) and

\[ \xi = \text{arcscin} \left( \frac{\delta}{\sqrt{v_1(\cdot) + v_2(\cdot) + \delta}} \right) \]

We applied this to\( \left(z_1(\cdot), w_1(\cdot)\right) \in \mathcal{S}(\epsilon^2 + 1, \epsilon^2 + 1)\) for \(\lambda = 2, \) and \(\lambda = 10.14.\) We simulated with and without \(\xi \) added to \(u_w\).

For the disturbance bound \( \delta > 0.21, \) we switched to

\[ u_w = \text{arcscin} \left( \frac{\delta^2}{\sqrt{v_1(\cdot) + v_2(\cdot) + \delta}} \right) \]

11. Conclusions

The PVTOL aircraft dynamics is a benchmark model that is of continuing ongoing research interest.

We developed a new bounded tracking feedback design that gives UGAS and ULES for a large class of reference trajectories.

Combined with the Do-Jiang-Pan observer design, our feedback applies when the velocity measurements are unavailable.

Our feedbacks give ISS performance to actuator disturbances for any a priori bound on the admissible disturbances.

Our proofs used a new bounded backstepping method which we anticipate being useful for other models in feedforward form.

12. References