Bounded Tracking Controllers and Robustness Analysis for UAVs

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Nonlinear Dynamical Systems and Applications IV
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Uncontrolled UAV Model

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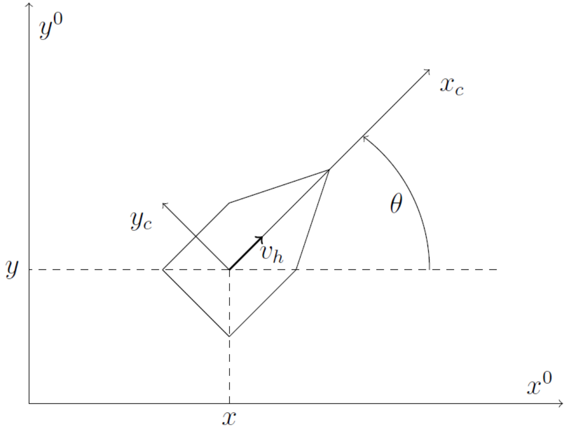

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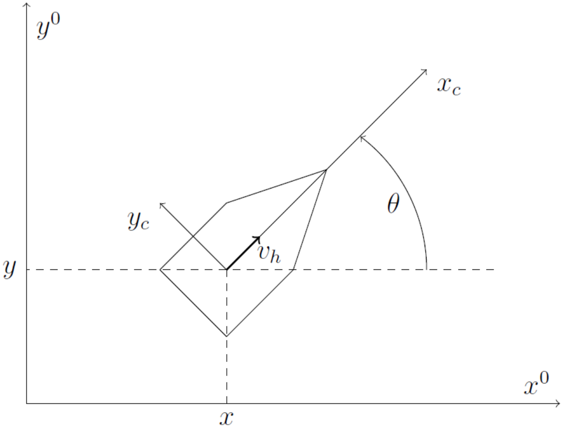

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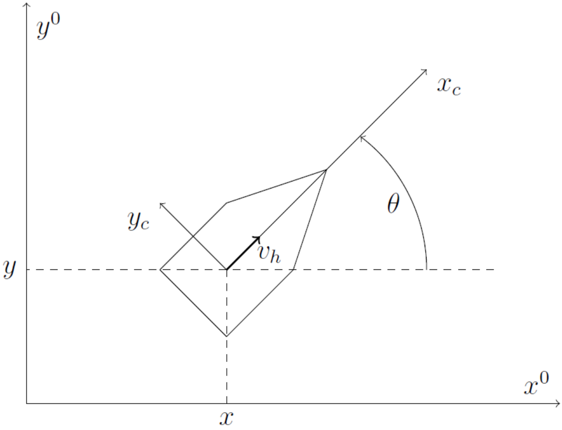

$$v_h = \begin{bmatrix} v \\ 0 \end{bmatrix} \qquad v_h^o = \begin{bmatrix} (v_h^o)_x \\ (v_h^o)_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v \\ 0 \end{bmatrix}$$

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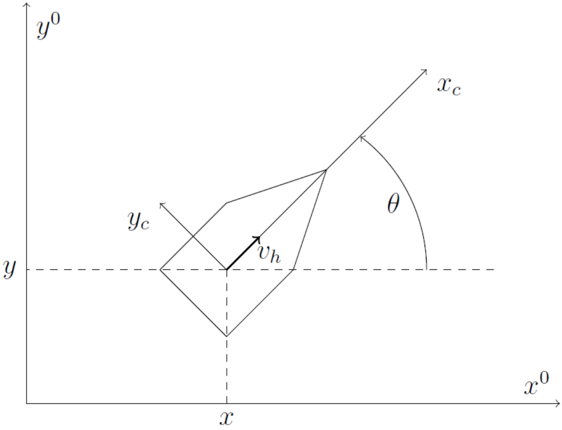

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix}$$

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Our Goal: Tracking with input-to-state stability with respect to disturbances under controller amplitude and rate constraints.

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This generalizes uniform global asymptotic stability to systems

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We show ISS and iISS properties with respect to $\mu = (\delta, \Delta)$.

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Definition: A C^2 function $\mathcal{R}_* = (x_*, y_*, \theta_*, v_*) : \mathbb{R} \to \mathbb{R}^3 \times (0, \infty)$ is called a trackable reference trajectory provided

1. x_*, y_*, $\dot{\theta}_*$, $\ddot{\theta}_*$, v_*, and $\dot{v}_*$ are bounded,
2. $\dot{x}_*(t) = v_*(t)\cos(\theta_*(t))$ and $\dot{y}_*(t) = v_*(t)\sin(\theta_*(t))$ hold for all $t \in \mathbb{R}$, and
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Consequence of Trackability: There are constants $c_0 > 0$ and $T > 0$ such that $\int_t^{t+T} [\dot{\theta}_*(s)]^2 ds \geq c_0$ for all $t \in \mathbb{R}$.

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Tracking Dynamics:

$$\left\{ \begin{aligned} \dot{\tilde{\psi}} &= -\dot{\theta}_*(t)\tilde{\xi} + \alpha_\theta[\tilde{\xi} + \xi_*(t)][\tilde{\theta} - \theta_N - \Delta] \\ \dot{\tilde{\xi}} &= \dot{\theta}_*(t)\tilde{\psi} + \tilde{v} - \alpha_\theta[\tilde{\psi} + \psi_*(t)][\tilde{\theta} - \theta_N - \Delta] \\ \dot{\tilde{\theta}} &= \alpha_\theta(-\tilde{\theta} + \theta_N + \Delta) \\ \dot{\tilde{v}} &= \alpha_v(-\tilde{v} + v_N + \delta) \end{aligned} \right. \tag{TD}$$

Theorem (Gruszka-M-Mazenc, TAC'12)

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Key Features: iISS Lyapunov functions. Controls do not depend on v and satisfy certain amplitude and rate constraints. $||v_N|| \leq k/\{\sqrt{2}\alpha_v\}$ and $||\theta_N|| \leq \sqrt{2}k \max\{||\xi_*||, ||\psi_*||\}$.

Control Amplitude and Rate Constraints

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Let $[\underline{\theta}_r, \bar{\theta}_r]$ and $[\underline{v}_r, \bar{v}_r]$ be the desired rate envelopes.

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Let $\varepsilon > 0$ be a constant and $[\underline{v}_a, \bar{v}_a]$ be the desired v_c envelope.

Assume that $\underline{v}_a + \varepsilon < v_*(t) + \dot{v}_*(t)/\alpha_v < \bar{v}_a - \varepsilon$ holds for all t. We can choose the constant $k > 0$ small enough such that $\underline{v}_a < v_c(t, \mathcal{E}(t)) < \bar{v}_a$ along all trajectories, and similarly for θ_c.

Let $[\underline{\theta}_r, \bar{\theta}_r]$ and $[\underline{v}_r, \bar{v}_r]$ be the desired rate envelopes.

Assume that $\underline{v}_r + \varepsilon < \dot{v}_*(t) + \ddot{v}_*(t)/\alpha_v < \bar{v}_r - \varepsilon$ holds for all t.

Control Amplitude and Rate Constraints

$$\begin{aligned} v_c(t,\mathcal{E}) &= v_N(\mathcal{E}) + v_*(t) + \dot{v}_*(t)/\alpha_v \\ \theta_c(t,\mathcal{E}) &= \theta_N(t,\mathcal{E}) + \theta_*(t) + \dot{\theta}_*(t)/\alpha_\theta \end{aligned}$$

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Assume that $\underline{v}_r + \varepsilon < \dot{v}_*(t) + \ddot{v}_*(t)/\alpha_v < \bar{v}_r - \varepsilon$ holds for all t.

For each constant $B > 0$, we can find a constant $\bar{K}(B)$ such that if $|(\tilde{\theta}(t_0), \tilde{v}(t_0))| \leq B$ and $k \in (0, \bar{K}(B))$ both hold, then $\underline{v}_r < \dot{v}_c(t, \mathcal{E}(t)) < \bar{v}_r$ along all trajectories, and similarly for $\dot{\theta}_c$.

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- We also aim to extend our work to coordinated control of uncertain UAVs under time delays in the controls.