Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

Michael Malisoff, Roy P. Daniels Professor of Mathematics at Louisiana State University

Sponsors: AFOSR, NSF/DMS, NSF/EPCN
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4 November 2015

What Do We Mean By Control Systems?

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t-\tau)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. $\delta : [0, \infty) \to \mathcal{D}$ is (nonstochastic) uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
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γ_i's are 0 at 0, strictly increasing, and unbounded. $\gamma_i \in \mathcal{K}_\infty$.

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When $\tau = 0$, a system is ISS iff it has an ISS LF (Sontag-Wang).

Michael Malisoff awarded two collaborative research grants from the NSF Directorate for Engineering

Submitted by kasten on Wed, 2014-08-27 14:21

Roy Paul Daniels Professor **Michael Malisoff** will serve as Lead Principal Investigator for two collaborative projects from the US National Science Foundation Directorate for Engineering. The first project "**Robustness of Networked Model Predictive Control Satisfying Critical Timing Constraints**," focuses on resolving contentions in a class of communication networks that are common in automobiles and other real-time control applications, and is joint with the Georgia Institute of Technology School of Electrical and Computer Engineering. The second project is "**Designs and Theory of State-Constrained Nonlinear Feedback Controls for Delay and Partial Differential Equation Systems**" and covers control designs for classes of ordinary and hyperbolic partial differential equations that arise in oil production and rehabilitation engineering, and is joint with the UC San Diego Department of Mechanical and Aerospace Engineering.

Print

LSU

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Collaborators: Fumin Zhang (GT) and Miroslav Krstic (UCSD)

PhD Students: S. Koga, R. Sizemore, J. Weston, N. Yao

Total 3-Year Budget: $871,000 (LSU Portion: $380,928)

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My Joint Work with F. Mazenc and M. de Queiroz

We solved the tracking and parameter identification problem for

$$\begin{cases} \dot{x} = f(\xi) \\ \dot{z}_i = g_i(\xi) + k_i(\xi)\theta_i + \psi_i u_i, \quad i = 1, 2, \ldots, s\,. \end{cases} \tag{3}$$

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Main PE Assumption: positive definiteness of the matrices

$$\mathcal{M}_i = \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \ \in \ \mathbb{R}^{(p_i+1)\times(p_i+1)}, \ 1 \leq i \leq s, \tag{4}$$

where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for $i = 1, 2, \ldots, s$.

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- We know v_f and a strict LF $V : [0, \infty) \times \mathbb{R}^{r+s} \to [0, \infty)$ for

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such that $-\dot{V}$ and V have positive definite quadratic lower bounds near 0, and V and v_f also have period T.

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- There are known positive constants θ_M, $\underline{\psi}$ and $\overline{\psi}$ such that

$$\underline{\psi} \; < \; \psi_i \; < \; \overline{\psi} \quad \text{and} \quad |\theta_i| \; < \; \theta_M \tag{6}$$

  for each $i \in \{1, 2, \ldots, s\}$. Known directions for the ψ_i's.

Dynamic Feedback

The estimator evolves on $\{\prod_{i=1}^{s}(-\theta_M, \theta_M)^{p_i}\} \times (\underline{\psi}, \overline{\psi})^s$.

$$\begin{cases} \dot{\hat{\theta}}_{i,j} &= (\hat{\theta}_{i,j}^2 - \theta_M^2)\varpi_{i,j}, \quad 1 \leq i \leq s\,, 1 \leq j \leq p_i \\ \dot{\hat{\psi}}_i &= \left(\hat{\psi}_i - \underline{\psi}\right)\left(\hat{\psi}_i - \overline{\psi}\right)\Upsilon_i, \quad 1 \leq i \leq s \end{cases} \tag{7}$$

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Barrier terms ensure that $\underline{\psi}<\hat{\psi}_i(t)<\overline{\psi}$ and $|\hat{\theta}_{i,j}(t)|<\theta_M$.

Augmented Error Dynamics

Tracking error: $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R)$
Parameter estimation errors: $\widetilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\widetilde{\psi}_i = \psi_i - \hat{\psi}_i$

$$\left\{ \begin{array}{rcl} \dot{\tilde{x}} & = & f(\tilde{\xi} + \xi_R(t)) - f(\xi_R(t)) \\ \dot{\tilde{z}}_i & = & v_{f,i}(t, \tilde{\xi}) + k_i(\tilde{\xi} + \xi_R(t))\widetilde{\theta}_i \\ & & +\widetilde{\psi}_i u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}), \;\; 1 \le i \le s \\ \dot{\widetilde{\theta}}_{i,j} & = & -\left(\hat{\theta}^2_{i,j} - \theta^2_M\right)\varpi_{i,j}, \; 1 \le i \le s \, , 1 \le j \le p_i \\ \dot{\widetilde{\psi}}_i & = & -\left(\hat{\psi}_i - \underline{\psi}\right)\left(\hat{\psi}_i - \overline{\psi}\right)\Upsilon_i, \; 1 \le i \le s \, . \end{array} \right. \qquad \text{(AED)}$$

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$$\begin{array}{rcl} \mathcal{Y} & = & \mathbb{R}^{r+s} \times \left(\prod_{i=1}^{s} \left\{\prod_{j=1}^{p_i} (\theta_{i,j} - \theta_M, \theta_{i,j} + \theta_M)\right\}\right) \\ & & \times\left(\prod_{i=1}^{s} \left(\psi_i - \overline{\psi}, \psi_i - \underline{\psi}\right)\right). \end{array}$$

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We build a strict LF for the augmented error dynamics for $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $\mathcal{Y}$.

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We start with this nonstrict barrier type LF on $\mathcal{Y}$:

$$V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\widetilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \mathrm{d}m + \sum_{i=1}^{s} \int_0^{\widetilde{\psi}_i} \frac{m}{(\psi_i - m - \underline{\psi})(\overline{\psi} - \psi_i + m)} \mathrm{d}m \, .$$

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We build a strict LF for the augmented error dynamics for $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $\mathcal{Y}$.

We start with this nonstrict barrier type LF on $\mathcal{Y}$:

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We transform V_1 into the desired strict LF for (AED).

Our Transformation

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Theorem: We can construct a function $\mathcal{L} \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) = \mathcal{L}\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Omega}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \;, \qquad (10)$$

$$\text{where} \quad \overline{\Omega}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) = -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) + \frac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \;, \qquad (11)$$

$$\begin{aligned} \alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) &= \begin{bmatrix} \tilde{\theta}_i\psi_i - \theta_i\tilde{\psi}_i \\ \widetilde{\psi}_i \end{bmatrix}, \text{ and} \\ \Omega_i(t) &= \int_{t-T}^{t}\int_m^t \lambda_i^\top(s)\lambda_i(s)\mathrm{d}s\,\mathrm{d}m \;, \end{aligned} \qquad (12)$$

is a strict LF for (AED) on its state space $\mathcal{Y}$, so (AED) is UGAS.

Application: Marine Robots (with Georgia Tech)

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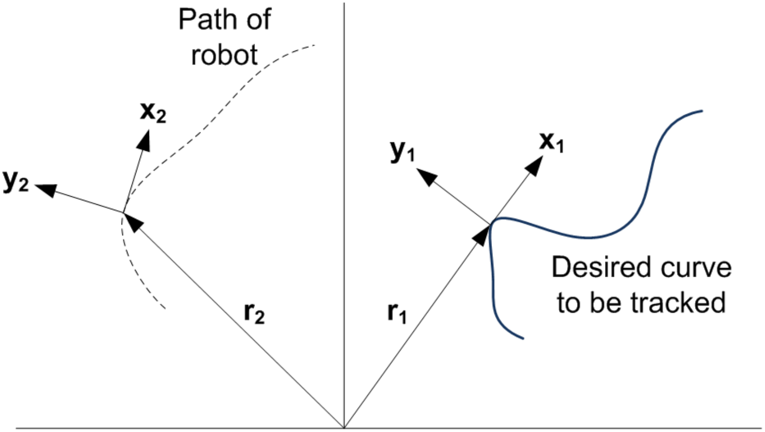

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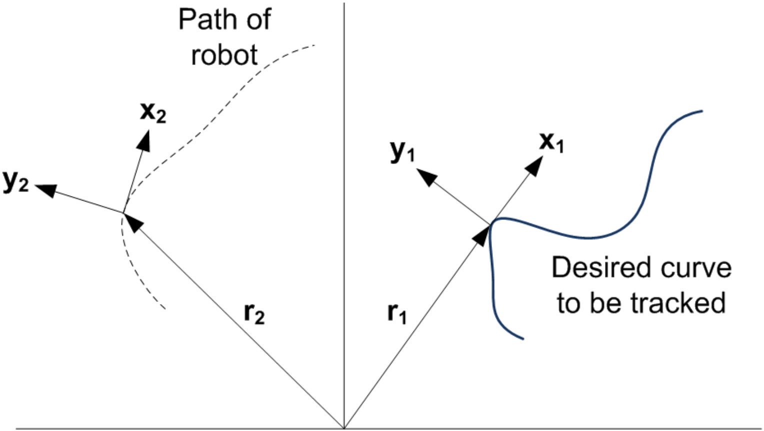

$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

Curve Tracking Dynamics

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Zhang et al, IEEE CDC, '04: Steering control

$$\begin{cases} \dot{\rho} = -\sin(\phi) \\ \dot{\phi} = \frac{\kappa \cos(\phi)}{1+\kappa\rho} - u_0, \quad (\rho, \phi) \in (0, \infty) \times (-\pi/2, \pi/2) \end{cases} \tag{13}$$

$$u_0 = \frac{\kappa \cos(\phi)}{1+\kappa\rho} - h'(\rho)\cos(\phi) + \mu \sin(\phi) \ , \ \kappa = \text{curvature} \tag{14}$$

$$h(\rho) = \alpha \left\{ \rho + \frac{\rho_0^2}{\rho} - 2\rho_0 \right\}, \ \ \rho_0 = \text{desired value for } \rho \tag{15}$$

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$$\text{New}: \ U(\rho,\phi) = -h'(\rho)\sin(\phi) + \frac{1}{\mu}\int_0^{V(\rho,\phi)} \Gamma_0(m)\mathrm{d}m \tag{17}$$

Our Robustly Forwardly Invariant Hexagons

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We used U to prove ISS of the $(\rho - \rho_0, \phi)$ system, where

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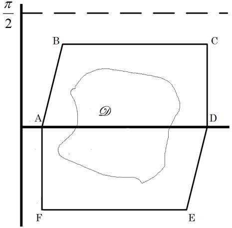

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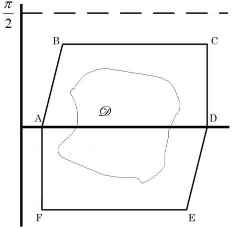

View the curve tracking state space $\mathcal{Y} = (0, \infty) \times (-\pi/2, \pi/2)$ as a union of hexagonal regions $H_1 \subseteq H_2 \subseteq \ldots H_i \subseteq \ldots$.

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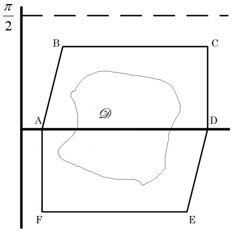

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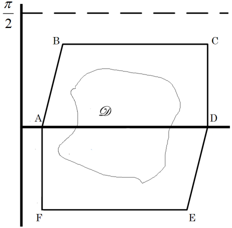

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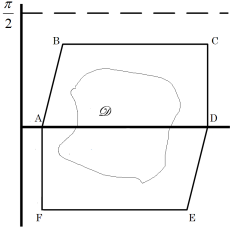

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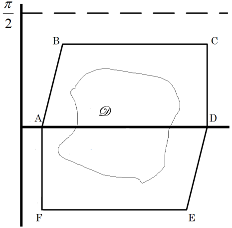

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$\Delta_{*i} = \min\{|h'(\rho)\cos(\phi)| : (\rho, \phi)^\top \in \mathrm{AB} \cup \mathrm{ED}\}$

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Our Adaptive Robust Tracking Control

$$\begin{cases} \dot{\rho} & = & -\sin(\phi) \\ \dot{\phi} & = & \frac{\kappa\cos(\phi)}{1+\kappa\rho} + K[u+\delta] \end{cases} \tag{19}$$

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$$\begin{cases} \dot{\tilde{q}}_1 & = & -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 & = & \frac{\kappa\cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K}{\tilde{K}+K}u_0 - K\delta \\ \dot{\tilde{K}} & = & -(\tilde{K}+K-c_{\min})(c_{\max}-\tilde{K}-K)\frac{\partial U}{\partial \phi}\frac{u_0}{\tilde{K}+K} \end{cases} \tag{20}$$

for $(\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K)$ on each set in our sequence of hexagonal regions that fill $\mathcal{Y} = (0, \infty) \times (-\pi/2, \pi/2)$.

Our Summer 2011 Field Work at Grand Isle, LA

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20 days of field work off Grand Isle.

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Georgia Tech Savannah Robotics Team. Joint with F. Zhang.

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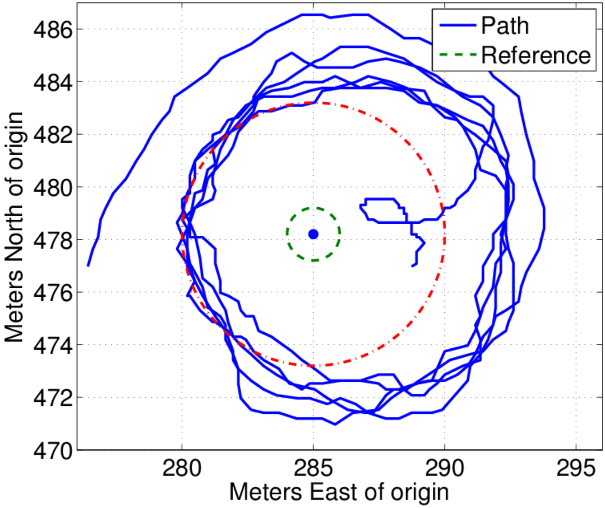
(Loading Video...)

Circle Tracking by ASV Victoria



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Conclusions

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A promising research direction is to study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.