Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

Michael Malisoff, Roy P. Daniels Professor of Mathematics at Louisiana State University

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North Carolina State University Mathematics Talk
4 November 2015
What Do We Mean By Control Systems?

\[ Y'(t) = F(t, Y(t), u(t, Y(t - \tau)), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}. \] (1)

\( \mathcal{Y} \subseteq \mathbb{R}^n \). \( \delta : [0, \infty) \rightarrow \mathcal{D} \) is (nonstochastic) uncertainty. \( \mathcal{D} \subseteq \mathbb{R}^m \).

The vector \( \Gamma \) is constant but unknown. \( \tau \) is a constant delay.
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\[
Y'(t) = G(t, Y(t), Y(t - \tau), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}, \tag{2}
\]

where \( G(t, Y(t), Y(t - \tau), \Gamma, d) = F(t, Y(t), u(t, Y(t - \tau)), \Gamma, d) \).

**Problem:** Given a trajectory \( Y_r \), specify \( u \) and a dynamics for an estimate \( \hat{\Gamma} \) of \( \Gamma \) such that the dynamics for the augmented error \( \mathcal{E}(t) = (Y(t) - Y_r(t), \Gamma - \hat{\Gamma}(t)) \) satisfies ISS with respect to \( \delta \).
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Flight control, electrical and mechanical engineering, etc.
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**Persistent excitation.** Annaswamy, Narendra, Teel.
What is Input-to-State Stability (or ISS)?

ISS (Sontag, ’89) generalizes uniform global asymptotic stability.
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\[ \mathcal{E}'(t) = G(t, \mathcal{E}(t), \mathcal{E}(t - \tau), \Gamma), \quad \mathcal{E}(t) \in \mathcal{Y} \quad \text{(Σ)} \]

\[ |\mathcal{E}(t)| \leq \gamma_1 \left( e^{t_0 - t} \gamma_2(|\mathcal{E}|_{[t_0 - \tau, t_0]}) \right) \quad \text{(UGAS)} \]

\( \gamma_i \)'s are 0 at 0, strictly increasing, and unbounded. \( \gamma_i \in \mathcal{K}_\infty \).
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Find \( \gamma_i \)'s by building special strict Lyapunov functions (LFs).
What is Input-to-State Stability (or ISS)?

ISS (Sontag, ’89) generalizes uniform global asymptotic stability.

\[ \dot{\mathcal{E}}(t) = G(t, \mathcal{E}(t), \mathcal{E}(t - \tau), \Gamma), \quad \mathcal{E}(t) \in \mathcal{Y} \tag{\Sigma} \]

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When \(\tau = 0\), a system is ISS iff it has an ISS LF (Sontag-Wang).
Michael Malisoff awarded two collaborative research grants from the NSF Directorate for Engineering

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Roy Paul Daniels Professor Michael Malisoff will serve as Lead Principal Investigator for two collaborative projects from the US National Science Foundation Directorate for Engineering. The first project "Robustness of Networked Model Predictive Control Satisfying Critical Timing Constraints," focuses on resolving contentions in a class of communication networks that are common in automobiles and other real-time control applications, and is joint with the Georgia Institute of Technology School of Electrical and Computer Engineering. The second project is "Designs and Theory of State-Constrained Nonlinear Feedback Controls for Delay and Partial Differential Equation Systems" and covers control designs for classes of ordinary and hyperbolic partial differential equations that arise in oil production and rehabilitation engineering, and is joint with the UC San Diego Department of Mechanical and Aerospace Engineering.
What is the Value Added by Your Research?

For many systems, we design controls that ensure ISS under the delays $\tau$ and uncertainties $\delta$ that prevail in engineering. Interconnect the systems with dynamics for estimators $\hat{\Gamma}(t)$ that converge to $\Gamma$, and then use $\hat{\Gamma}(t)$ in $u$, instead of $\Gamma$. Under state constraints $Y$, choose sets $S \subseteq Y$ to find maximal perturbation sets $D$ the system can tolerate without leaving $S$. To handle delays $\tau$, we use Lyapunov-Krasovskii or Razumikhin functions, or predictive or other dynamic controls. Active magnetic bearings, bioreactors, brushless DC motors, heart rate controllers, human pointing motions, marine robots, microelectromechanical relays, neuromuscular electrical stimulation, underactuated ships, unmanned air vehicles,..
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My Joint Work with F. Mazenc and M. de Queiroz

We solved the tracking and parameter identification problem for

\[
\begin{align*}
\dot{x} &= f(\xi) \\
\dot{z}_i &= g_i(\xi) + k_i(\xi)\theta_i + \psi_i u_i, \quad i = 1, 2, \ldots, s.
\end{align*}
\]

\(\xi = (x, z) \in \mathbb{R}^{r+s}.\)
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\]

(3)

\[\xi = (x, z) \in \mathbb{R}^{r+s}. \quad (\theta, \psi) = (\theta_1, \ldots, \theta_s, \psi_1, \ldots, \psi_s) \in \mathbb{R}^{p_1 + \ldots + s + ps + s}.\]

The \(C^2\) reference trajectory \(\xi_R = (x_R, z_R)\) is assumed to have some period \(T > 0\) and satisfy \(\dot{x}_R(t) = f(\xi_R(t))\) for all \(t \geq 0\).
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**Main PE Assumption:** positive definiteness of the matrices

\[\mathcal{M}_i = \int_0^T \lambda_i^\top(t)\lambda_i(t) \, dt \in \mathbb{R}^{(p_i+1) \times (p_i+1)}, \quad 1 \leq i \leq s,\]  

(4)

where \(\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))\) for \(i = 1, 2, \ldots, s.\)
Two Other Key Assumptions

\[\begin{align*}
\dot{X} &= f(X, Z) + \xi R(t) - f(\xi R(t)) \\
\dot{Z} &= v_f(t, X, Z)
\end{align*}\] (5)

such that

\[\begin{align*}
-\dot{V} &\text{ and } V \text{ have positive definite quadratic lower bounds near 0, and}
\end{align*}\]

\[V \text{ and } v_f \text{ also have period } T.\]

Key: Reduces the LF construction problem to (5).

\[\begin{align*}
\text{There are known positive constants } \theta_M, \psi, \psi &\text{ such that } \\
\psi &< \psi_i < \psi \text{ and } |\theta_i| < \theta_M (6)
\end{align*}\]

for each \(i \in \{1, 2, \ldots, s\}.\) Known directions for the \(\psi_i\)'s.
Two Other Key Assumptions

▶ We know $v_f$ and a strict LF $V : [0, \infty) \times \mathbb{R}^{r+s} \rightarrow [0, \infty)$ for

$$
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\dot{X} &= f((X, Z) + \xi_R(t)) - f(\xi_R(t)) \\
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$$

such that $-\dot{V}$ and $V$ have positive definite quadratic lower bounds near 0, and $V$ and $v_f$ also have period $T$.

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**Key:** Reduces the LF construction problem to (5).

- There are known positive constants \( \theta_M, \underline{\psi} \) and \( \overline{\psi} \) such that

\[
\underline{\psi} < \psi_i < \overline{\psi} \quad \text{and} \quad |\theta_i| < \theta_M
\]

(6)

for each \( i \in \{1, 2, \ldots, s\} \). Known directions for the \( \psi_i \)'s.
Dynamic Feedback

The estimator evolves on \( \prod_{i=1}^{s}(-\theta_M, \theta_M)^{p_i} \times (\psi, \overline{\psi})^s \).

\[
\begin{align*}
\dot{\hat{\theta}}_{i,j} &= (\hat{\theta}_{i,j}^2 - \theta_M^2)\varpi_{i,j}, \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\dot{\hat{\psi}}_i &= (\hat{\psi}_i - \psi) (\hat{\psi}_i - \overline{\psi}) \gamma_i, \quad 1 \leq i \leq s
\end{align*}
\]

(7)

Here \( \hat{\theta}_i = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_i}) \) for \( i = 1, 2, \ldots, s \),

\[
\varpi_{i,j} = -\frac{\partial V}{\partial \tilde{z}_i}(t, \tilde{\xi}) k_{i,j}(\tilde{\xi} + \xi_R(t))
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Dynamic Feedback

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\]

(8)

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u_{f,i}(t, \tilde{\xi}) - g_i(\xi) - k_i(\xi) \dot{\hat{\theta}}_i + \dot{z}_{R,i}(t) \]

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(9)

Estimator and feedback can only depend on things we know.
**Dynamic Feedback**

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\[
\begin{cases}
\dot{\hat{\theta}}_{i,j} = (\hat{\theta}^{2}_{i,j} - \theta^{2}_{M})\varpi_{i,j}, & 1 \leq i \leq s, 1 \leq j \leq p_{i} \\
\dot{\hat{\psi}}_{i} = (\hat{\psi}_{i} - \underline{\psi}) (\hat{\psi}_{i} - \overline{\psi}) \gamma_{i}, & 1 \leq i \leq s
\end{cases}
\]

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Here \( \hat{\theta}_{i} = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_{i}}) \) for \( i = 1, 2, \ldots, s \),

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\]

\[
\gamma_{i} = -\frac{\partial V}{\partial \tilde{z}_{i}}(t, \tilde{\xi}) u_{i}(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) .
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Estimator and feedback can only depend on things we know.

**Barrier terms** ensure that \( \underline{\psi} < \hat{\psi}_{i}(t) < \overline{\psi} \) and \( |\hat{\theta}_{i,j}(t)| < \theta_{M} \).
Augmented Error Dynamics

Tracking error: \( \tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R) \)

Parameter estimation errors: \( \tilde{\theta}_i = \theta_i - \hat{\theta}_i \) and \( \tilde{\psi}_i = \psi_i - \hat{\psi}_i \)

\[
\begin{align*}
\dot{\tilde{x}} &= f(\tilde{\xi} + \xi_R(t)) - f(\xi_R(t)) \\
\dot{\tilde{z}}_i &= v_{f,i}(t, \tilde{\xi}) + k_i(\tilde{\xi} + \xi_R(t))\tilde{\theta}_i \\
&\quad + \tilde{\psi}_i u_i(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}), \quad 1 \leq i \leq s \\
\dot{\tilde{\theta}}_{i,j} &= -\left(\hat{\theta}_{i,j}^2 - \theta_M^2\right)\varpi_{i,j}, \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\dot{\tilde{\psi}}_i &= -\left(\hat{\psi}_i - \psi\right)\left(\hat{\psi}_i - \bar{\psi}\right)\gamma_i, \quad 1 \leq i \leq s.
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\begin{align*}
\dot{\tilde{x}} &= f(\tilde{\xi} + \xi_R(t)) - f(\xi_R(t)) \\
\dot{\tilde{z}}_i &= v_{f,i}(t, \tilde{z}) + k_i(\tilde{\xi} + \xi_R(t))\tilde{\theta}_i \\
&\quad + \tilde{\psi}_i u_i(t, \tilde{z}, \tilde{\theta}, \hat{\psi}), \quad 1 \leq i \leq s \\
\dot{\tilde{\theta}}_{i,j} &= -\left(\hat{\theta}_{i,j}^2 - \theta_i^2\right)\tilde{\omega}_{i,j}, \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\dot{\tilde{\psi}}_i &= -\left(\hat{\psi}_i - \psi\right)\left(\hat{\psi}_i - \overline{\psi}\right)\tilde{\gamma}_i, \quad 1 \leq i \leq s.
\end{align*}
\]

(AED)

\[
\mathcal{Y} = \mathbb{R}^{r+s} \times \left(\prod_{i=1}^{s} \left\{\prod_{j=1}^{p_i} (\theta_{i,j} - \theta_M, \theta_{i,j} + \theta_M)\right\}\right) \\
\times \left(\prod_{i=1}^{s} (\psi_i - \overline{\psi}, \psi_i - \psi)\right).
\]
Stabilization Analysis

We build a strict LF for the augmented error dynamics for $E = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi^R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $Y$.

We start with this nonstrict barrier type LF on $Y$:

$V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = V(t, \tilde{\xi}) + s\sum_{i=1}^{p_i} \sum_{j=1}^{m_\theta} (m - \theta_i, j)^2 d\theta_i + s\sum_{i=1}^{p_i} \int_{\psi_i}^\infty m_\psi \psi (\psi - \psi_i + m_\psi) d\psi$.

On $Y$, $\dot{V}_1 \leq -W(\tilde{\xi})$ for some positive definite function $W$.

We transform $V_1$ into the desired strict LF for (AED).
Stabilization Analysis

We build a strict LF for the augmented error dynamics for
\( E = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( \mathcal{Y} \).
Stabilization Analysis

We build a strict LF for the augmented error dynamics for 
\( \mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( \mathcal{Y} \).

We start with this nonstrict barrier type LF on \( \mathcal{Y} \):

\[
V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_{0}^{\tilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \, dm \\
+ \sum_{i=1}^{s} \int_{0}^{\tilde{\psi}_i} \frac{m}{(\psi_i - m - \bar{\psi})(\bar{\psi} - \psi_i + m)} \, dm.
\]
Stabilization Analysis

We build a strict LF for the augmented error dynamics for \( \mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( \mathcal{Y} \).

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+ \sum_{i=1}^{s} \int_{0}^{\tilde{\psi}_i} \frac{m}{(\psi_i - m - \psi)(\psi - \psi_i + m)} dm.
\]

On \( \mathcal{Y} \), \( \dot{V}_1 \leq -W(\tilde{\xi}) \) for some positive definite function \( W \).
Stabilization Analysis

We build a strict LF for the augmented error dynamics for \( \mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( \mathcal{Y} \).

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We transform \( V_1 \) into the desired strict LF for (AED).
Our Transformation
Our Transformation

**Theorem:** We can construct a function $\mathcal{L} \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\#(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = \mathcal{L}(V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi})) + \sum_{i=1}^{s} \Omega_i(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) , \quad (10)$$

where

$$\Omega_i(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = -\tilde{z}_i \lambda_i(t) \alpha_i(\tilde{\theta}_i, \tilde{\psi}_i)$$

$$+ \frac{1}{T\psi} \alpha_i^\top(\tilde{\theta}_i, \tilde{\psi}_i) \Omega_i(t) \alpha_i(\tilde{\theta}_i, \tilde{\psi}_i) , \quad (11)$$

$$\alpha_i(\tilde{\theta}_i, \tilde{\psi}_i) = \begin{bmatrix} \tilde{\theta}_i \psi_i - \theta_i \tilde{\psi}_i \\ \tilde{\psi}_i \end{bmatrix} , \quad \text{and}$$

$$\Omega_i(t) = \int_{t-T}^{t} \int_m \lambda_i^\top(s) \lambda_i(s) ds \ dm , \quad (12)$$

is a strict LF for (AED) on its state space $\mathcal{Y}$, so (AED) is UGAS.
Application: Marine Robots (with Georgia Tech)
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$\rho = |r_2 - r_1|$, $\phi = \text{angle between } x_1 \text{ and } x_2$, $\cos(\phi) = x_1 \cdot x_2$
Application: Marine Robots (with Georgia Tech)

\[ \rho = |\mathbf{r}_2 - \mathbf{r}_1|, \quad \phi = \text{angle between } \mathbf{x}_1 \text{ and } \mathbf{x}_2, \quad \cos(\phi) = \mathbf{x}_1 \cdot \mathbf{x}_2 \]
Curve Tracking Dynamics

\[ \dot{\rho} = -\sin(\phi) \]

\[ \dot{\phi} = \kappa \cos(\phi) + \frac{1}{\kappa} \rho - u_0, \quad (\rho, \phi) \in (0, \infty) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \]  

(13)

\[ u_0 = \kappa \cos(\phi) + \kappa \rho - h'(\rho) \cos(\phi) + \mu \sin(\phi), \quad \kappa = \text{curvature} \]  

(14)

\[ h(\rho) = \alpha \left\{ \rho + \rho^2 \rho_0 - \frac{1}{2} \right\} \]  

(15)

\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho) \]  

(16)

\[ U(\rho, \phi) = -h'(\rho) \sin(\phi) + \frac{1}{\mu} \int_0^{\Gamma_0(\mu)} V(\rho, \phi) \, dm \]  

(17)
Curve Tracking Dynamics

Zhang et al, IEEE CDC, ’04: Steering control

\[
\begin{align*}
\dot{\rho} &= -\sin(\phi) \\
\dot{\phi} &= \frac{\kappa \cos(\phi)}{1 + \kappa \rho} - u_0, \quad (\rho, \phi) \in (0, \infty) \times (-\pi/2, \pi/2)
\end{align*}
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u_0 = \frac{\kappa \cos(\phi)}{1 + \kappa \rho} - h'(\rho) \cos(\phi) + \mu \sin(\phi), \quad \kappa = \text{curvature}
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h(\rho) = \alpha \left\{ \rho + \frac{\rho_0^2}{\rho} - 2\rho_0 \right\}, \quad \rho_0 = \text{desired value for } \rho
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New: \[U(\rho, \phi) = -h'(\rho) \sin(\phi) + \frac{1}{\mu} \int_0^{\Gamma_0(m)} \Gamma_0(m) \, dm \tag{17}\]
Our Robustly Forwardly Invariant Hexagons
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We used $U$ to prove ISS of the $(\rho - \rho_0, \phi)$ system, where

$$
\dot{\rho} = -\sin(\phi), \quad \dot{\phi} = h'(\rho) \cos(\phi) - \mu \sin(\phi) + \delta
$$

(18)

and $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$, on certain forward invariant sets $H_i$. 

Tight Disturbance Bound: Choose any $\delta_{*i} \in (0, \min\{\Delta_{*i}, \Delta_{**i}\})$. 

$$
\Delta_{*i} = \min\{|h'(\rho) \cos(\phi)| : (\rho, \phi)^\top \in AB \cup ED\}
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View the curve tracking state space $\mathcal{Y} = (0, \infty) \times (-\pi/2, \pi/2)$ as a union of hexagonal regions $H_1 \subseteq H_2 \subseteq \ldots H_i \subseteq \ldots$. 
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\]
Our Adaptive Robust Tracking Control

\[\begin{align*}
\dot{\rho} &= -\sin(\phi) \\
\dot{\phi} &= \frac{\kappa \cos(\phi)}{1 + \kappa \rho} + Ku + \delta
\end{align*}\]  

(19)

\(\xi = (\rho, \phi), \theta_i = 0, \psi_i = K, f(\xi) = -\sin(\phi), g_i(\xi) = \frac{\kappa \cos(\phi)}{1 + \kappa \rho}\)
Our Adaptive Robust Tracking Control

\[
\begin{aligned}
\dot{\rho} &= -\sin(\phi) \\
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\end{aligned}
\]  

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Take \( u = -u_0/\hat{K} \).
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Take \(u = -u_0/\hat{K}\). We proved ISS for the dynamics

\[
\begin{align*}
\dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\
\dot{\tilde{q}}_2 &= \frac{\kappa \cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K}{\tilde{K}+K} u_0 - K\delta \\
\dot{\tilde{K}} &= -(\tilde{K} + K - c_{\text{min}})(c_{\text{max}} - \tilde{K} - K) \frac{\partial U}{\partial \phi} \frac{u_0}{\tilde{K}+K}
\end{align*}
\]

for \((\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K)\) on each set in our sequence of hexagonal regions that fill \(\mathcal{Y} = (0, \infty) \times (-\pi/2, \pi/2)\).
Our Summer 2011 Field Work at Grand Isle, LA

20 days of field work off Grand Isle.
Search for oil spill remnants.
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Circle Tracking by ASV Victoria


Conclusions

Adaptive nonlinear controllers are useful for many engineering control systems with delays and uncertainties. Curve tracking controllers for autonomous marine vehicles are important for monitoring water quality, especially after oil spills. Our controls identify parameters and are adaptive and robust to the perturbations and delays that arise in field work. We can prove these properties using ISS, dynamic extensions, and Lyapunov-Krasovskii functionals. We used our controls on student built marine robots to map residual crude oil from the Deepwater Horizon spill. A promising research direction is to study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.
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