Model-Based Nonlinear Control of the Human Heart Rate During Treadmill Exercising

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Model has been validated with human subjects. Unlike conventional linear models, it captures peripheral effects and is suitable for long duration exercise.

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is globally exponentially stable to zero, i.e., there are constants $c_i > 0$ so that $|\tilde{x}(t)| \le c_1 e^{-c_2 t} |\tilde{x}(0)|$ for all trajectories of (1).

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We always assume that there is a constant $\varepsilon \in (0, 1]$ such that

$$\frac{a_1 a_3}{a_2 a_4} > P_\varepsilon \stackrel{\text{def}}{=} \max\left\{\frac{1+\varepsilon}{\varepsilon}, \sup_{t\geq 0} \frac{1+b(1+\{1+\varepsilon\}x_{1r}(t))e^{-x_{1r}(t)}}{[1+be^{-\{1+\varepsilon\}x_{1r}(t)}][1+be^{-x_{1r}(t)}]}\right\} \qquad \text{(SA)}$$

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Cheng et al. use the Levenberg-Marquardt method to estimate the a_i's.

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The nonlinear controller

$$u_c(x,t) = \sqrt{\max\left\{0, u_r(t)^2 - \left(1 + \frac{R(\tilde{x}_1, t)}{P_\varepsilon}\right)\tilde{x}_2\right\}}, \tag{4}$$

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Proof: Take $V(\tilde{x}_1, \tilde{x}_2) = \frac{1}{2}\tilde{x}_1^2 + \frac{k}{2}\tilde{x}_2^2$, where $k = \frac{a_2}{a_4 P_\varepsilon}$.

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Proposition. The $(\tilde{x}, \overline{x})$ dynamics in closed loop with (6) is globally exponentially stable to the origin.

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Proof: Take $V^\sharp(\tilde{x}, \bar{x}) = V(\tilde{x}) + \bar{L}|\bar{x}|^2$ for a big enough $\bar{L} > 0$.

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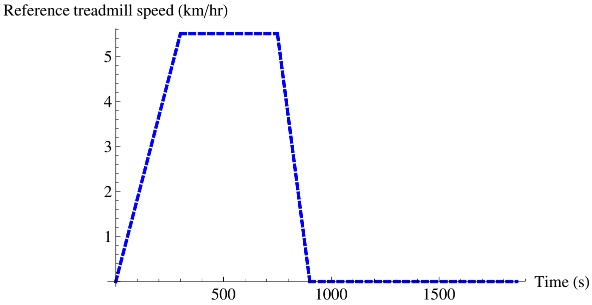

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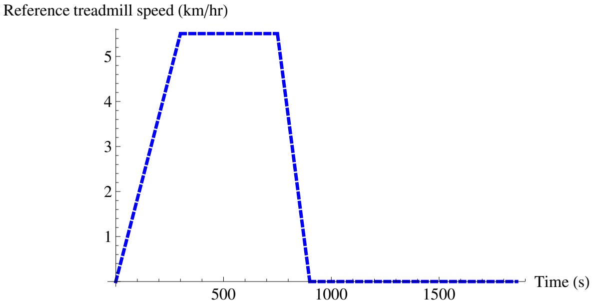

The resulting x_{1r} satisfies (SA) with $\varepsilon = 0.5$ so our results apply.

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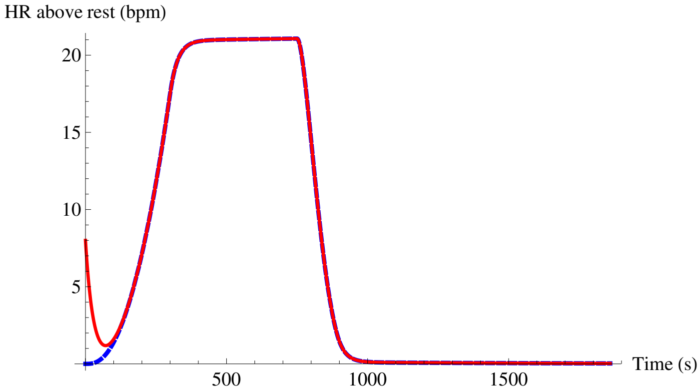

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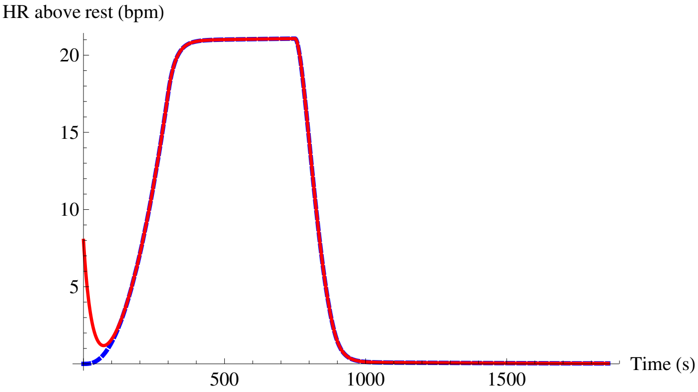

x_{1r} (blue and dashed) and state x_1 (red and solid).

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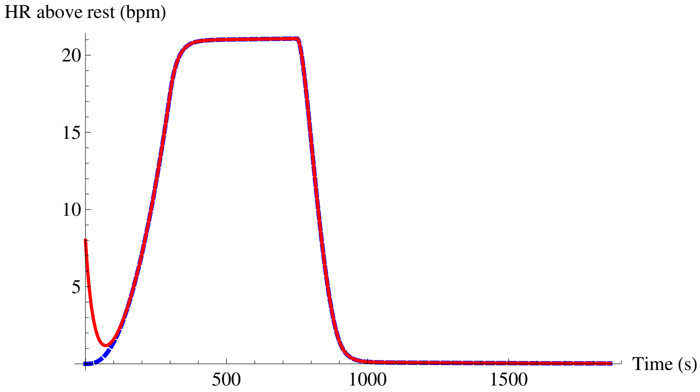

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Initial state: $x(0) = (2, 0)$.

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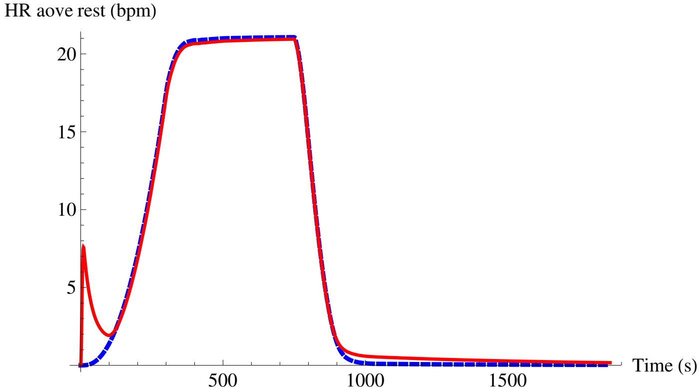

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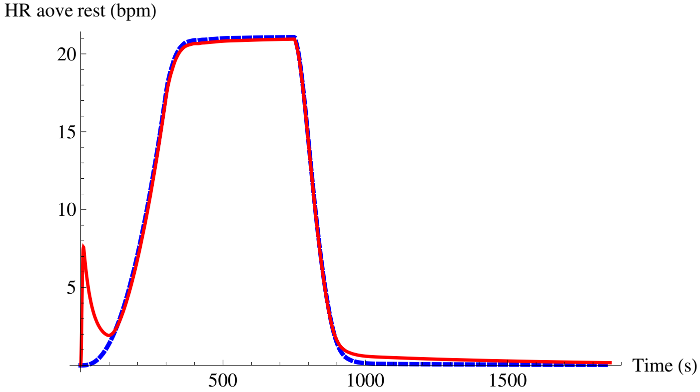

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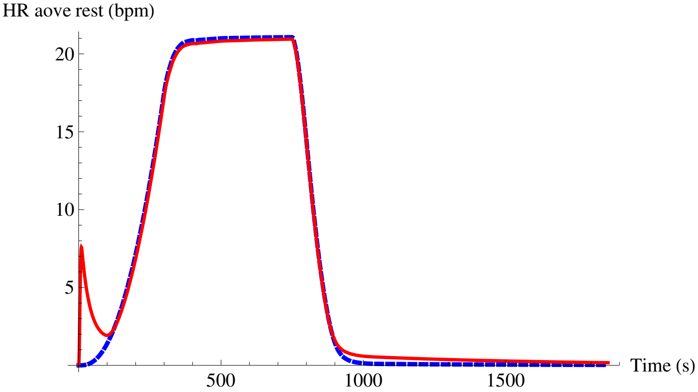

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Initial states: $x(0) = (0.01, 0.05)$, $\hat{x}(0) = (2, 0.3)$.

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Theorem: For each constant $\bar{\delta} > 0$, we can find constants $\bar{c}_i > 0$ depending on $\bar{\delta}$ so that along all trajectories of (8) for all measurable functions $\mathbf{d} : [0, \infty) \to [-\bar{\delta}, \bar{\delta}]$, we have $|(\tilde{x}(t), \bar{x}(t))| \leq \bar{c}_1 |(\tilde{x}(0), \bar{x}(0))| e^{-\bar{c}_2 t} + \bar{c}_3 |\mathbf{d}|_{[0,t]}$ for all $t \geq 0$.

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Proof: Pick $\bar{L} > 0$ so that $V^\sharp$ is an ISS Lyapunov function.

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- For complete proofs, see [FM, MM, and MdQ, “Tracking control and robustness analysis for a nonlinear model of human heart rate during exercise," Automatica, accepted.]