Adaptive Tracking and Estimation for Nonlinear Control Systems

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Joint with Frédéric Mazenc and Marcio de Queiroz
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- Novelty: Our explicit global strict Lyapunov function for the $Y = (\Psi - \hat{\Psi}, x - x_R)$ dynamics. It gave input-to-state stability with respect to additive time-varying uncertainties δ on Ψ.

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Both are shown by constructing specific kinds of strict Lyapunov functions for $\dot{Y} = \mathcal{G}(t, Y, 0)$.

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$$\mathcal{P}_i \stackrel{\text{def}}{=} \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \in \mathbb{R}^{(p_i+1)\times(p_i+1)}, \tag{9}$$

  where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for each i.

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$$\begin{array}{rcl} \varpi_{i,j} & = & -\frac{\partial V}{\partial \tilde{z}_i}(t,\tilde{\xi})k_{i,j}\big(\tilde{\xi}+\xi_R(t)\big) \;\text{ and} \\ \mho_i & = & -\frac{\partial V}{\partial \tilde{z}_i}(t,\tilde{\xi})u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi})\,. \end{array} \tag{13}$$

$$u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi}) \;=\; \frac{v_{f,i}(t,\tilde{\xi})-g_i(\xi)-k_i(\xi)\cdot\hat{\theta}_i+\dot{z}_{R,i}(t)}{\hat{\psi}_i} \tag{14}$$

The estimator and feedback can only depend on things we know.

Stabilization Analysis

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Theorem: We can construct $K \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \doteq K\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Upsilon}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \quad , \tag{15}$$

$$\begin{aligned} \text{where} \quad \overline{\Upsilon}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) &= -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \\ &\quad + \tfrac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \quad , \end{aligned} \tag{16}$$

$$\lambda_i(t) = \big(k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t))\big) \quad , \tag{17}$$

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- We covered systems with unknown control gains including brushless DC motors turning mechanical loads.
- It would be useful to extend to cover models that are not affine in θ, feedback delays, and output feedbacks.