Lyapunov Functions and Robustness Analysis under Matrosov Conditions with an Application to Biological Systems

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OUTLINE

- Objectives and Motivation
- Definitions and Assumptions
- Main Theorem for $\dot{x} = f(x)$
- Biotechnological Example
- Simulations
- Conclusions and Summary

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Lyapunov Functions under Matrosov Conditions: Global strict ones constructed in Mazenc-Nesic MCSS'07. Very general conditions. Teel-Loria et al, TAC'05: important sufficient conditions for stability.

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- Bounded below by positive definite quadratics near 0
- Only require a non-strict *positive definite* V s.t. $\dot{V} \leq 0$

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INPUT-TO-STATE STABILITY

ISS of (Σ): $\exists \beta \in \mathcal{KL}$, $\mathcal{M} \in M(D)$ and $\gamma \in \mathcal{K}_\infty$ s.t. for all $t \geq t_o \geq 0$, $x_o \in D$, and $\delta \in \mathcal{L}_\infty(\mathcal{C})$,

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M(D): all continuous functions $\mathcal{M} : D \to [0, \infty)$ s.t. (A) $\mathcal{M}(0) = 0$ and (B) $\mathcal{M}(x) \to +\infty$ as $x \to \partial D$ or $|x| \to +\infty$ while remaining in D.

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ASSUMPTIONS

Assumption 1: $\exists j \geq 2$; functions $V_i : D \to \mathbb{R}$, $\mathcal{N}_i : D \to \mathbb{R}_{\geq 0}$, and $\phi_i : \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$; and constants $a_i \in (0, 1]$ such that these Matrosov Conditions hold:

(a)

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(c) for $i = 2, \ldots, j$ and all $x \in D$,
$\nabla V_i(x) f(x) \leq -\mathcal{N}_i(x) + \phi_i(V_1(x)) \sum_{l=1}^{i-1} \mathcal{N}_l^{a_i}(x) V_1^{1-a_i}(x)$,

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(d) V_1 is positive definite on D.

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(ii) There exist functions $p_i : \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}$ s.t. $|V_i(x)| \leq p_i(V_1(x))V_1(x) \; \forall x \in D$, $i \in \{2, \ldots, j\}$.

MAIN THEOREM for $\dot{x} = f(x)$

Theorem: One can build explicit functions $k_l, \Omega_l \in \mathcal{K}_\infty \cap C^1$ such that

$$S(x) = \sum_{l=1}^{j} \Omega_l \left(k_l(V_1(x)) + V_l(x)\right)$$

satisfies $S(x) \geq V_1(x)$ and $\nabla S(x) f(x) \leq -\frac{1}{4}\rho(V_1(x))V_1(x)$ $\forall x \in D$.

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Assumptions: $\exists$ positive $\underline{\Delta}$ and $\bar{\Delta}$ s.t. $s\bar{\Delta}(s,x) \geq r(s,x) \geq xs\underline{\Delta}(s,x)$. $\exists \gamma_m, \gamma_M > 0$ s.t. $\gamma_* := k/[\lambda(s_{in} - s_*)] \in (\gamma_m, \gamma_M)$ & $k/[\lambda s_{in}] < \gamma_m$.

ADAPTIVE CONTROLLER

Augmented Dynamics: Setting $v_* = s_{in} - s_*$, $x_* = \frac{v_*}{k\alpha}$, $\tilde{x} = x - x_*$, $\tilde{s} = s - s_*$, $\tilde{\gamma} = \gamma - \gamma_*$, and $u = \gamma y_1$ yields

$$\dot{\tilde{s}} = -\gamma\tilde{s} + \tilde{\gamma}v_*, \quad \dot{\tilde{x}} = \alpha\left[-\gamma\tilde{x} - \tilde{\gamma}x_*\right],$$

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& $V_2(\tilde{s}, \tilde{\gamma}) = -\tilde{s}\tilde{\gamma}$.

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SIMULATIONS

Noisy Error Dynamics: Using small bounds on the δ_i's, can prove ISS of

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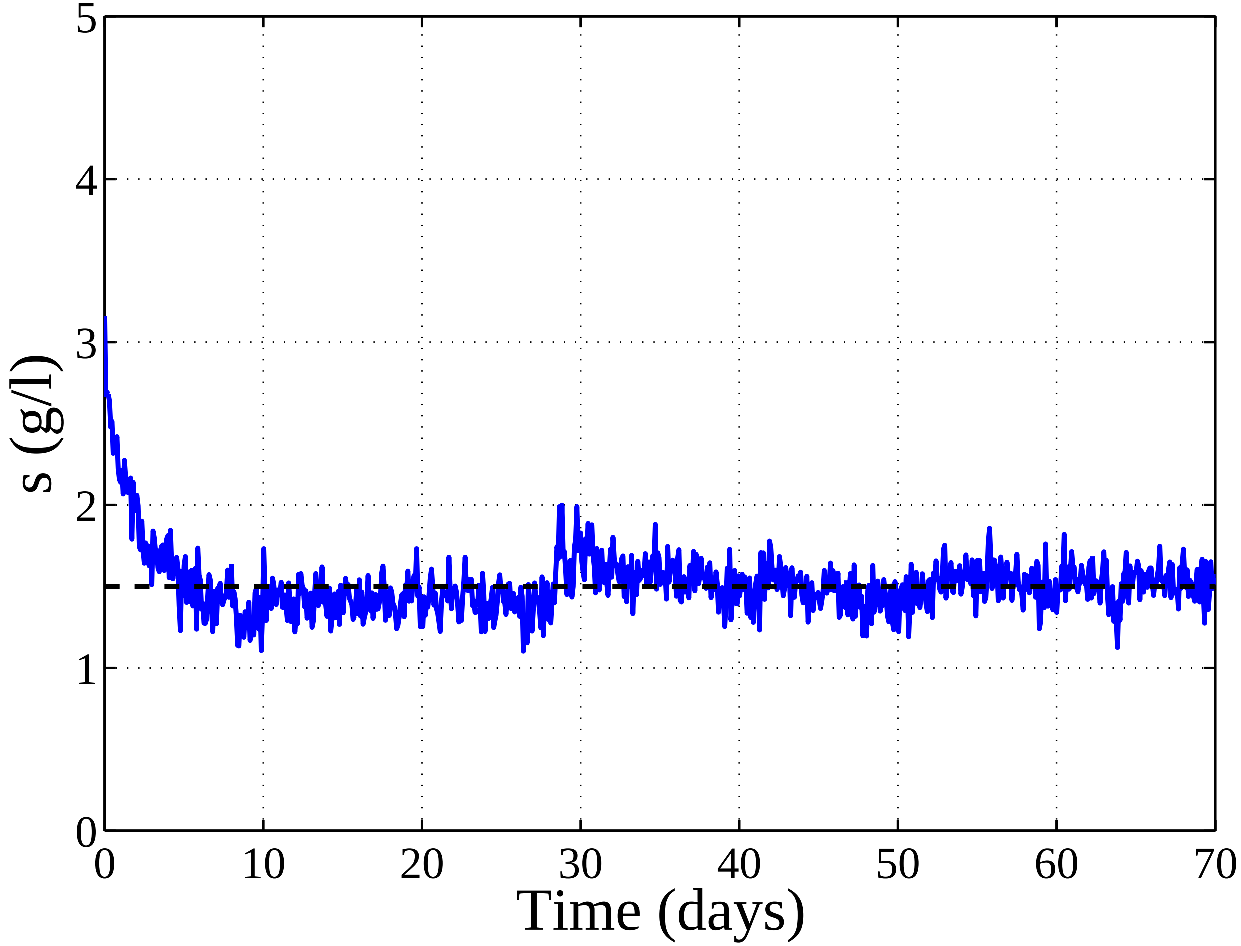
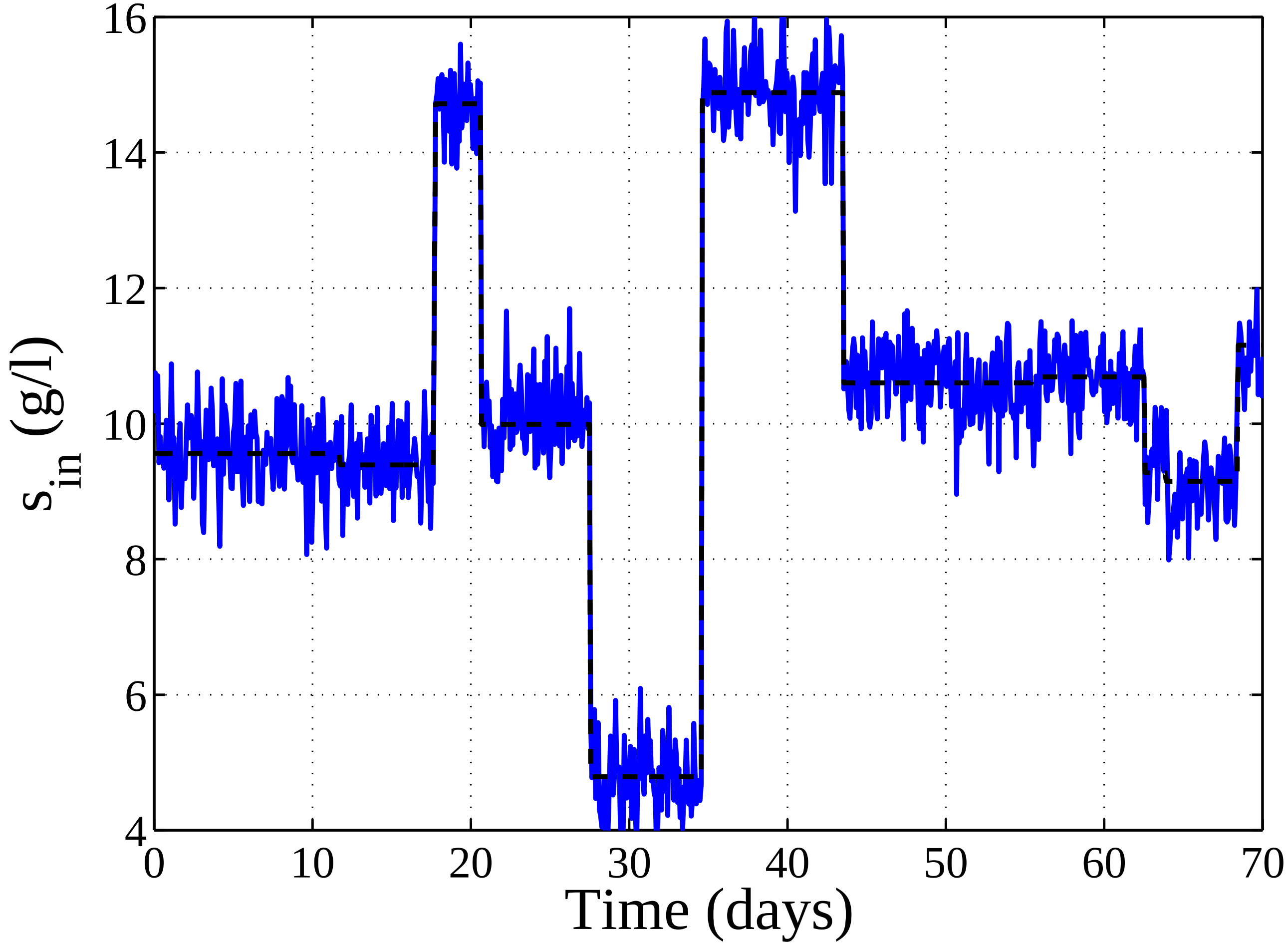
Simulations: Used parameters from Bernard et al., *Water Sci Tech*, 2006 with $s_* = 1.5$. δ_2 =1% white noise. δ_1 and δ_3 of s.d. 0.5 g/l and 0.1 g/l.

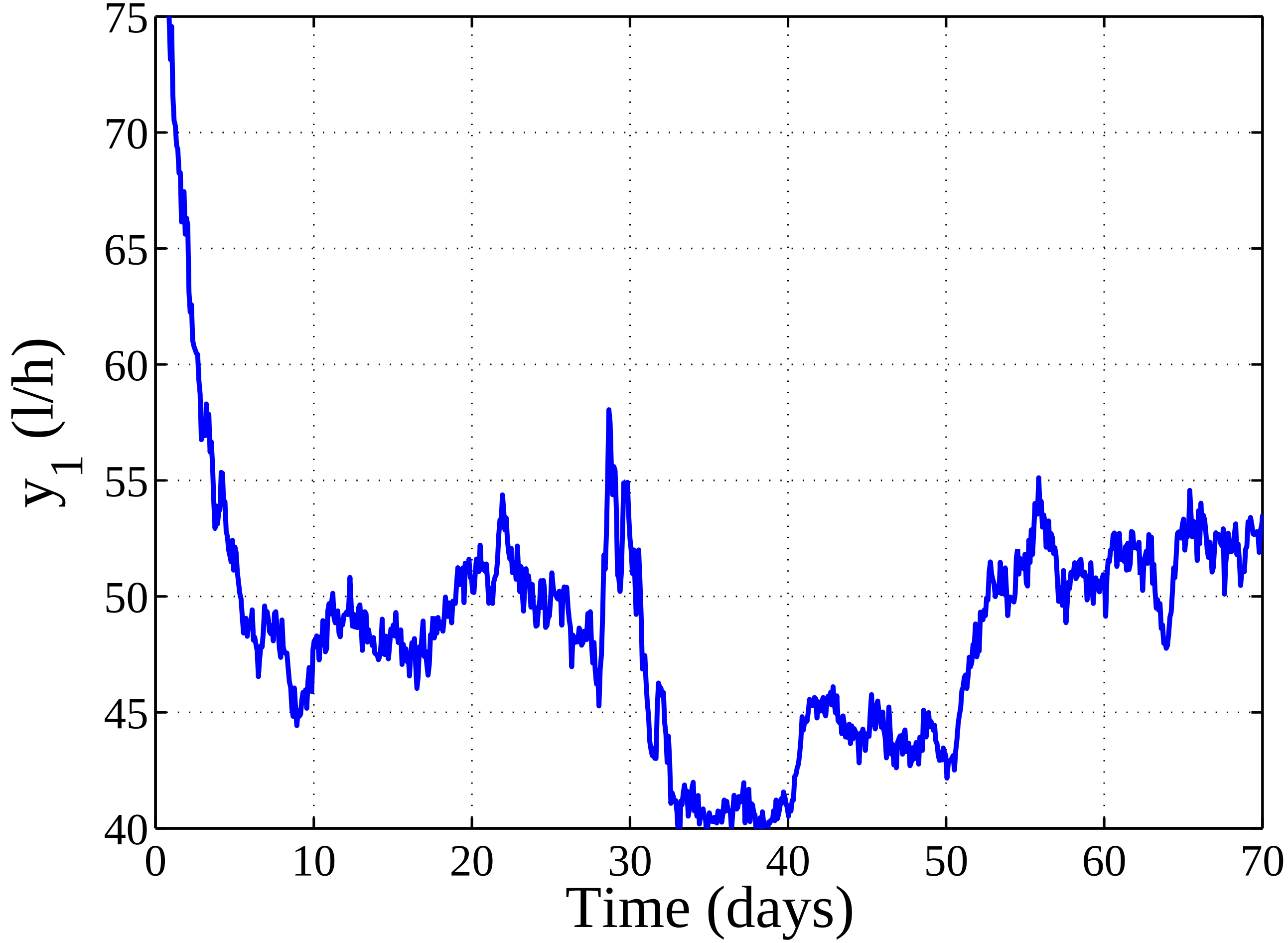
SIMULATIONS (cont'd)



SIMULATIONS (cont'd)



SIMULATIONS (cont'd)



OUTLINE

- Objectives and Motivation
- Definitions and Assumptions
- Main Theorem for $\dot{x} = f(x)$
- Idea of Proof
- Biotechnological Example
- Simulations
- Conclusions and Summary

CONCLUSIONS

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- Extensions to time-varying systems and applications to systems satisfying LaSalle conditions can also be shown.
- We used our constructions to give new constructive ISS results for the noisy experimental anaerobic digester.
- For complete proofs, see [Mazenc, M., M. Malisoff, and O. Bernard, “A Simplified Design for Strict Lyapunov Functions under Matrosov Conditions,” *IEEE Trans. Automat. Control*, provisionally accepted.]

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