Stability Analysis of Switched Systems with Time-Varying Discontinuous Delays

Frederic Mazenc

Michael Malisoff

Hitay Ozbay

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- ▷ Briat, Fridman, Liberzon, Liu-Teel, Mancilla-Aguilar-Garcia,..

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$$\dot{x}(t) = f_{\sigma(t)}(t, x(t), x(t - \tau(t)), \delta(t)) \qquad (\Sigma)$$

on $\mathbb{R}^n$, with $\tau : [0, +\infty) \to [0, \tau_b]$ piecewise continuous and the switching times t_i for the switching signal $\sigma : [0, +\infty) \to \{1, ..., k\}$ admitting constants $\mathcal{T}_i$ such that $0 < \mathcal{T}_1 < t_{i+1} - t_i \leq \mathcal{T}_2$ for all i.

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We place certain growth or decay conditions on suitable functions V_i along solutions of the ith subsystem

$$\dot{x}(t) = f_{\sigma(t_i)}(t, x(t), x(t - \tau(t)), \delta(t)) \qquad (\Sigma_i)$$

on intervals $[t_i, t_{i+1})$ when σ is constant and where (Σ_i) operates.

Standing Assumptions

1: There are k absolutely continuous functionals V_i, real constants α_i, nonnegative constants β_i, a continuous function W, and functions $\chi_i \in \mathcal{K}_\infty$ and $\Gamma_i \in \mathcal{K}_\infty$ such that

$$\chi_1(|\phi(0)|) \le V_i(t,\phi) \le \chi_2(|\phi|_\infty) \qquad (G_g)$$

hold for all $\phi \in C_{\mathrm{in}}$, $t \in [0,+\infty)$, and $i \in \{1,2,\dots,k\}$ and

$$\begin{aligned} \dot{V}_{\sigma(t_i)}(t) \quad \le \quad & \alpha_{\sigma(t_i)} V_{\sigma(t_i)}(t,x_t) + \beta_{\sigma(t_i)} \sup_{\ell\in[t-\tau_b,t]} W(x(\ell)) \\ & + \Gamma_{\sigma(t_i)}(|\delta(t)|) \end{aligned} \qquad (D_g)$$

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2: There is a constant $\mu > 1$ such that $W(\phi(0)) \leq V_1(t,\phi)$ and $V_i(t,\phi) \leq \mu V_j(t,\phi)$ hold for all $\phi \in C_{\text{in}}$, all i and j, and all $t \geq 0$.

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Standing Assumptions and First Theorem

3: There are constants $T \geq \tau_b + \mathcal{T}_2$ and $\lambda > 0$ such that

$$\int_{t-T}^{t} \alpha_{\sigma(\ell)} \mathrm{d}\ell \leq -\lambda \tag{1}$$

holds for all $t \geq T$.

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4: With the choice $\bar{c} = \frac{1}{\mu-1}(\mu^{L+2} - 1) - L - 1$, we have

$$\mu^{L+1} e^{-\lambda} + \bar{c}\mu \sup_{t \geq T} \int_{t-T}^{t} \beta_{\sigma(s)} e^{\int_s^t \alpha_{\sigma(s)} \mathrm{d}\ell} \mathrm{d}s < 1, \tag{2}$$

where $L = \sup_{t \geq T} \text{Cardinality}\{t_i : t - T \leq t_i < t\}$.

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Theorem: Under the preceding assumptions, (Σ) is ISS on $\mathbb{R}^n$.

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▷ Mazenc, F., M. Malisoff, and S.-I. Niculescu. Stability and control design for time-varying systems with time-varying delays using a trajectory based approach. *SIAM Journal on Control and Optimization*, 55(1):533-556, 2017.

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Thank you for your attention!