Predictor-Based Tracking for Neuromuscular Electrical Stimulation

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Sponsored by NSF/ECCS/EPAS Program
Summary of International Journal of Robust and Nonlinear Control Paper

2014 SIAM Annual Meeting
MS113, Engineering Applications of Mathematics
Thursday July 10th, 4:30-4:55, Paper 2

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Our new control only needs sampled observations, allows any delay, and tracks position and velocity under a state constraint.

NMES on Leg Extension Machine

(Loading Video...)

Leg extension machine at Warren Dixon's NCR Lab at U of FL

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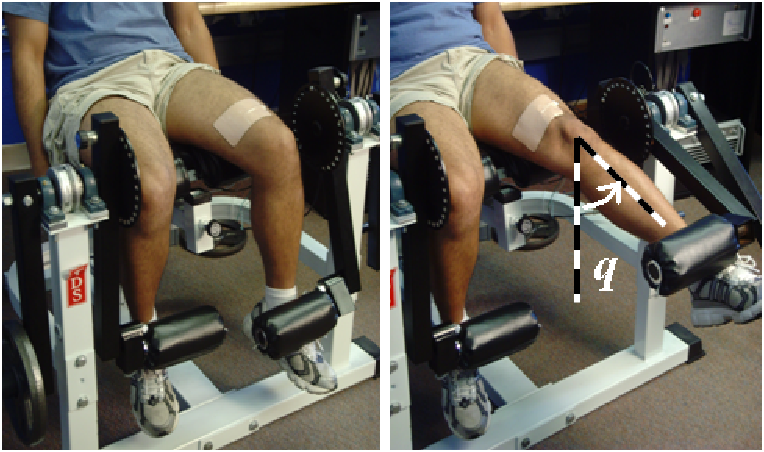

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Mathematical Model

$$\overbrace{J\ddot{q}}^{M_I(\ddot{q})} + \overbrace{b_1\dot{q} + b_2\tanh(b_3\dot{q})}^{M_v(\dot{q})} + \overbrace{k_1 q e^{-k_2 q} + k_3\tan(q)}^{M_e(q)}$$
$$+ \underbrace{\mathcal{M}gl\sin(q)}_{M_g(q)} = \mathcal{A}(q,\dot{q})\, v(t-\tau), \quad q \in (-\tfrac{\pi}{2}, \tfrac{\pi}{2}) \qquad (1)$$

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J and $\mathcal{M}$ are inertia and mass of the lower limb/machine, the b_i's and k_i's are positive damping and elastic constants, respectively, l is the distance between the knee joint and the center of the mass of the lower limb/machine, $\tau > 0$ is a delay.

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$$\max\{||\dot{q}_d||_\infty, ||v_d||_\infty, ||\dot{v}_d||_\infty\} < \infty \text{ and } ||q_d||_\infty < \tfrac{\pi}{2} \tag{4}$$

Voltage Potential Controller

$$v(t) = \frac{g_2(\zeta_d(t+\tau))v_d(t) - g_1(\zeta_d(t+\tau)+\xi(t)) + g_1(\zeta_d(t+\tau)) - (1+\mu^2)\xi_1(t) - 2\mu\xi_2(t)}{g_2(\zeta_d(t+\tau)+\xi(t))}$$

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and $\xi(T_i) = z_{N_i}$.

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and $\xi(T_i) = z_{N_i}$. The time-varying Euler iterations $\{z_k\}$ at each time T_i use measurements $(q(T_i), \dot{q}(T_i))$.

Voltage Potential Controller (continued)

Euler iterations used for control:

$$z_{k+1} = \Omega(T_i + kh_i, h_i, z_k; v) \ \text{ for } \ k = 0, ..., N_i - 1 \ , \ \text{where}$$

$$z_0 = \begin{pmatrix} \tan(q(T_i)) - \tan(q_d(T_i)) \\ \frac{\dot{q}(T_i)}{\cos^2(q(T_i))} - \frac{\dot{q}_d(T_i)}{\cos^2(q_d(T_i))} \end{pmatrix}, \quad h_i = \frac{\tau}{N_i} \ ,$$

and $\Omega : [0, +\infty)^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$\Omega(T, h, x; v) = \begin{bmatrix} \Omega_1(T, h, x; v) \\ \Omega_2(T, h, x; v) \end{bmatrix} \tag{5}$$

and the formulas

$$\begin{aligned} \Omega_1(T, h, x; v) &= x_1 + hx_2 \ \text{ and} \\ \Omega_2(T, h, x; v) &= x_2 + \zeta_{2,d}(T) + \int_T^{T+h} g_1(\zeta_d(s) + x)\mathrm{d}s \\ &\quad + \int_T^{T+h} g_2(\zeta_d(s) + x)\, v(s - \tau)\mathrm{d}s - \zeta_{2,d}(T + h). \end{aligned}$$

Our Tracking Theorem for NMES

For all positive constants τ and r, there exist a locally bounded function N, a constant $\omega \in (0, \mu/2)$ and a locally Lipschitz function C satisfying $C(0) = 0$ such that: For all sample times $\{T_i\}$ in $[0, \infty)$ such that $\sup_{i \geq 0}(T_{i+1} - T_i) \leq r$ and each initial condition, the solution $(q(t), \dot{q}(t), v(t))$ with

$$\begin{aligned} N_i &= N\left(\left|\left(\tan(q(T_i)), \frac{\dot{q}(T_i)}{\cos^2(q(T_i))}\right) - \zeta_d(T_i)\right| \right. \\ &\qquad \left. + ||v - v_d||_{[T_i - \tau, T_i]}\right) \end{aligned} \tag{6}$$

satisfies

$$\begin{aligned} &|q(t) - q_d(t)| + |\dot{q}(t) - \dot{q}_d(t)| + ||v - v_d||_{[t-\tau, t]} \\ &\leq \; e^{-\omega t} C\left(\frac{|q(0) - q_d(0)| + |\dot{q}(0) - \dot{q}_d(0)|}{\cos^2(q(0))} + ||v_0 - v_d||_{[-\tau, 0]}\right) \end{aligned}$$

for all $t \geq 0$.

Ideas from Proof

Our main lemma gives general conditions on systems of the form $\dot{x}(t) = f(t, x(t), u(t))$ that allow us to predict future states of the system, using an explicit Euler method with iterates

$$x_{i+1} = x_i + \int_{t_0+ih}^{t_0+(i+1)h} f(s, x_i, u(s))\mathrm{d}s, \quad 0 \le i \le N-1 \,, \qquad (7)$$

where $h = \frac{\tau}{N}$, $x_0 \in \mathbb{R}^n$, and $u : [t_0, t_0 + \tau) \to \mathbb{R}^m$ are given.

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The lemma builds functions A_i such that for any $\tau > 0$, $x_0 \in \mathbb{R}^n$, $t_0 \geq 0$, and measurable bounded function $u : [t_0, t_0 + \tau) \to \mathbb{R}^m$, the solution of $\dot{x}(t) = f(t, x(t), u(t))$, $x(t_0) = x_0$ satisfies

$$|x(t_0 + \tau) - x_N| \ \leq \ \frac{\tau A_1(|x_0|+||u||)}{N}\left(e^{\tau A_2(|x_0|+||u||)} - 1\right) \qquad (8)$$

for all $N \geq \tau A_3\left(|x_0| + ||u||\right)$.

First Simulation

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$\tau = 0.07\text{s}$, $\mathcal{A}(q,\dot{q}) = \bar{a}e^{-2q^2}\sin(q) + \bar{b}$

$$\begin{aligned} &J = 0.39\,\mathrm{kg\text{-}m^2/rad},\ b_1 = 0.6\,\mathrm{kg\text{-}m^2/(rad\text{-}s)},\ \bar{a} = 0.058,\\ &b_2 = 0.1\,\mathrm{kg\text{-}m^2/(rad\text{-}s)},\ b_3 = 50\,\mathrm{s/rad},\ \bar{b} = 0.0284,\\ &k_1 = 7.9\,\mathrm{kg\text{-}m^2/(rad\text{-}s^2)},\ k_2 = 1.681/\mathrm{rad},\\ &k_3 = 1.17\,\mathrm{kg\text{-}m^2/(rad\text{-}s^2)},\ \mathcal{M} = 4.38\,\mathrm{kg},\ l = 0.248\,\mathrm{m}. \end{aligned} \tag{10}$$

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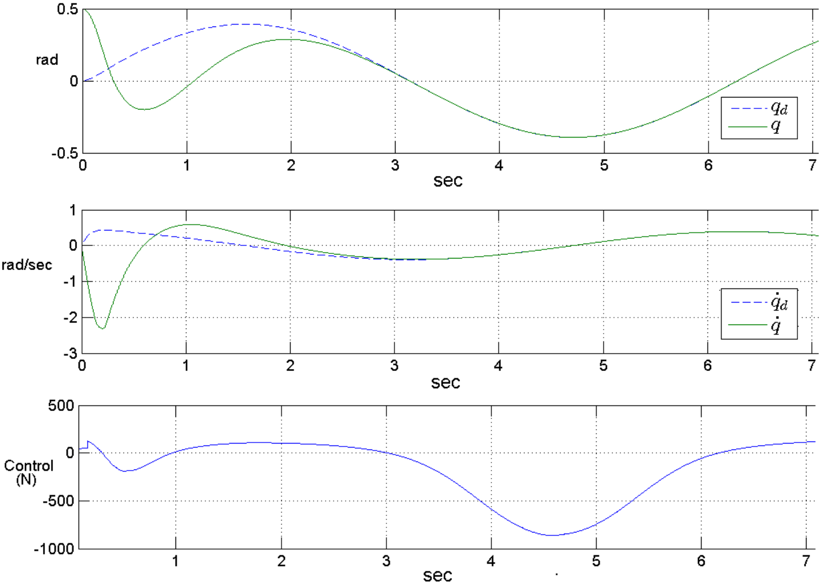
$N_i = N = 10$, and $T_{i+1} - T_i = 0.014\text{s}$, and $\mu = 2$.

First Simulation



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$$q_d(t) = \frac{\pi}{3}\left(1 - \exp\left(-3t\right)\right)\ \mathrm{rad}, \tag{13}$$

$q(0) = \frac{\pi}{18}$, $\dot{q}(0) = v_0(t) = 0$, $N_i = N = 10$, $T_{i+1} - T_i = 0.014$.

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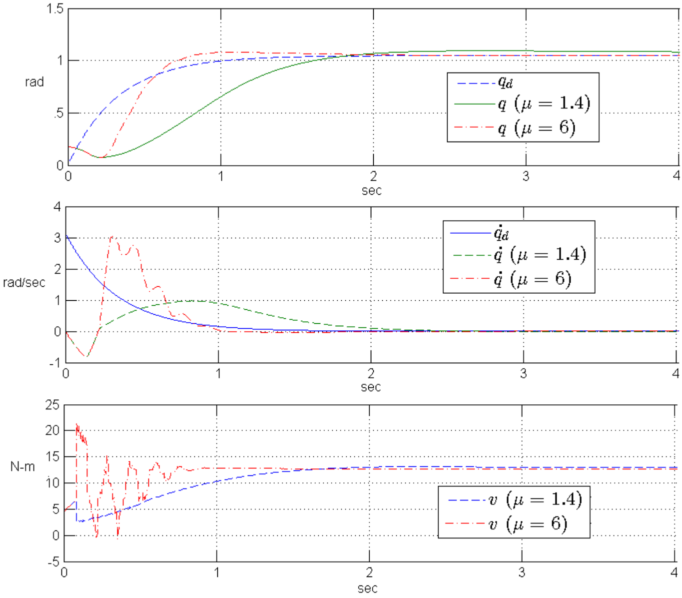
We used these mismatched parameters in the control:

$$\begin{aligned} &J' = 1.25J,\ \ b_1' = 1.2b_1,\ \ b_2' = 0.9b_2,\ \bar{a}' = 1.185\bar{a},\\ &b_3' = 0.85b_3,\ \ k_1' = 1.1k_1,\ \ k_2' = 0.912k_2,\ \bar{b}' = 0.98\bar{b},\\ &k_3' = 0.9k_3,\ \ \mathcal{M}' = 0.97\mathcal{M},\ \ \text{and}\ \ l' = 1.013l. \end{aligned} \tag{14}$$

Simulated Robustness Test



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Our control used a new numerical solution approximation method that covers many other time-varying models.

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Our new sampled predictive control design overcame these challenges and can track a large class of reference trajectories.

By incorporating the state constraint on the knee position, our control can help ensure patient safety for any input delay value.

Our control used a new numerical solution approximation method that covers many other time-varying models.

In future work, we hope to apply input-to-state stability to better understand the effects of uncertainties under state constraints.