Robustness of Adaptive Control for Three-Dimensional Curve Tracking under State Constraints: Effects of Scaling Control Terms

Michael Malisoff

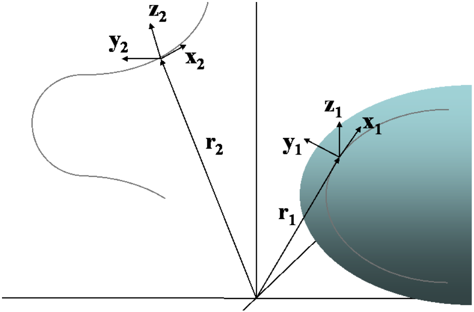
Robert Sizemore

Fumin Zhang

Variant of:

M. Malisoff and F. Zhang. Robustness of adaptive control under time delays for three-dimensional curve tracking. *SIAM Journal on Control and Optimization*, 53(4):2203-2236, 2015.

3D Curve Tracking by Unit Speed Robot



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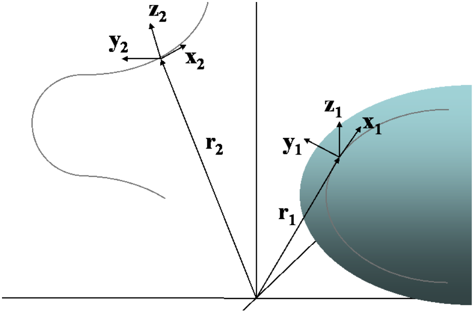

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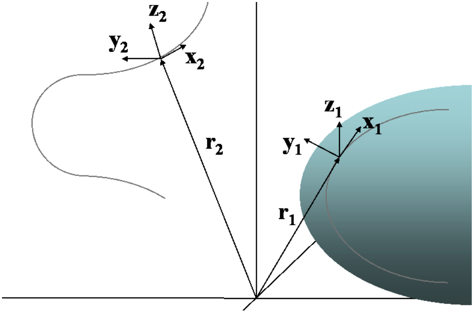

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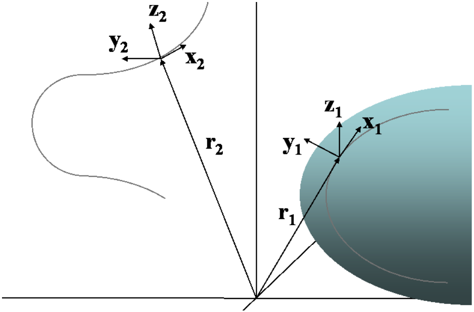

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Goal: Find u and v such that $|\mathbf{r_1}(t) - \mathbf{r_2}(t)| \to \rho_c$ for a desired $\rho_c > 0$ and $\mathbf{x_1} \cdot \mathbf{x_2} \to 1$, while compensating for additive and multiplicative control uncertainty, delays, and state constraints.

Our New Variables and Control Design

$(\rho_1, \rho_2) = ((\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{y}_1, (\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{z}_1)$ has desired value (ρ_{c1}, ρ_{c2}).

$\rho_c = |(\rho_{c1}, \rho_{c2})|$. Shape vars: $\varphi = \mathbf{x}_1 \cdot \mathbf{x}_2$, $\beta = \mathbf{y}_1 \cdot \mathbf{x}_2$, $\gamma = \mathbf{z}_1 \cdot \mathbf{x}_2$

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u &= a_1(\mathbf{x}_1 \cdot \mathbf{y}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{y}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{y}_2), \\
v &= a_1(\mathbf{x}_1 \cdot \mathbf{z}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{z}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{z}_2), \\
a_1 &= \mu,\ a_2 = -h_1'(\rho_1) + \tfrac{\alpha\kappa_n}{\varphi},\ a_3 = -h_2'(\rho_2) + \tfrac{\alpha\kappa_g}{\varphi},\ \text{and} \qquad (1) \\
h_i(\rho_i) &= \begin{cases} \bar{c}\left(\rho_i + \rho_{ci}^2/\rho_i - 2\rho_{ci}\right), & \rho_i \in (0, \rho_{ci}) \\ \frac{\bar{c}}{\rho_{ci}}(\rho_i - \rho_{ci})^2, & \rho_i \geq \rho_{ci} \end{cases}
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New State $Y = (\rho_1, \zeta, \rho_2, \theta)$ takes its values in $\mathcal{X}$, where $(\varphi, \beta, \gamma) = \big(\cos(\zeta)\cos(\theta), -\sin(\zeta)\cos(\theta), \sin(\theta)\big)$ and where $\mathcal{X} = (0, \infty) \times (-\pi/2, \pi/2) \times (0, \infty) \times (-\pi/2, \pi/2)$.

First Key Ingredient: Strict Lyapunov Function

$$
\begin{aligned}
\dot{\rho}_1 &= -\sin(\zeta)\cos(\theta) \\
\dot{\zeta} &= -\frac{1}{\cos^2(\theta)}\Big[\alpha\kappa_n \sin^2(\theta) - h_1'(\rho_1)\cos(\zeta)\cos(\theta) \\
&\quad + \alpha\kappa_g \sin(\theta)\sin(\zeta)\cos(\theta) + \mu\sin(\zeta)\cos(\theta)\Big] \\
\dot{\rho}_2 &= \sin(\theta) \\
\dot{\theta} &= \alpha\kappa_g \frac{\sin^2(\zeta)}{\cos(\zeta)} - h_2'(\rho_2)\cos(\theta) - \mu\cos(\zeta)\sin(\theta) \\
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Theorem (MZ, SICON'15): We can build a function $\mathcal{L}$ such that

$$U(Y) = -h_1'(\rho_1)\sin(\zeta)\cos(\theta) + h_2'(\rho_2)\sin(\theta) + \int_0^{V(Y)} \mathcal{L}(q)\mathrm{d}q$$

is a strict Lf for (2) for the equilibrium $\mathcal{E} = (\rho_{c1}, 0, \rho_{c2}, 0)$ on $\mathcal{X}$, where $V(Y) = -\ln(\cos(\theta)\cos(\zeta)) + h_1(\rho_1) + h_2(\rho_2)$.

Second Key Ingredient: Robust Forward Invariance

$$\begin{aligned} \dot{Y} &= \mathcal{F}(Y) + \left(0, \left(\frac{G}{\hat{G}} - 1\right)\mathcal{A}_1(Y) + \delta_1, 0, \left(\frac{G}{\hat{G}} - 1\right)\mathcal{A}_2(Y) + \delta_2\right) \\ \dot{\hat{G}} &= (g_{\max} - \hat{G})(\hat{G} - g_{\min})\frac{1}{\hat{G}}\left(\frac{\partial U}{\partial \zeta}(Y)\mathcal{A}_1(Y) + \frac{\partial U}{\partial \theta}(Y)\mathcal{A}_2(Y)\right) \end{aligned} \tag{3}$$

where $\mathcal{F}(Y)$ is the right side of (2), $G \in I_G \stackrel{def}{=} (g_{\min}, g_{\max})$ is the unknown control gain, and the right side of the Y subsystem is obtained by replacing the controls by $u/\hat{G}$ and $v/\hat{G}$.

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We built compact paired hexagons $\mathcal{S}_i$ containing $\mathcal{E}$ such that $\cup_{i=1}^{\infty}\mathcal{S}_i = \mathcal{X}$, and sequences $\{\bar{\delta}_{1i}\}$ and $\{\bar{\delta}_{2i}\}$, such that for all i:

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RFI: $\{Y(t, Y_0, \delta) : t \geq 0, Y_0 \in \mathcal{S}_i, \delta \in \mathcal{MEB}([0, \infty), \mathcal{D}_i)\} \subseteq \mathcal{S}_i$

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It scales the steering constant μ and the penalty functions h_i.

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Thank you for your attention!