Bounded Backstepping Control Designs for Time-Varying Systems

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$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{F}(t, x(t), z_1(t)) \\ \dot{z}_i(t) & = & z_{i+1}(t), \quad i \in \{1, 2, \ldots, k-1\} \\ \dot{z}_k(t) & = & u(t) + h(t, x(t), z(t)) \end{array} \right. \tag{1}$$

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After changes of variables and feedback for any constant $q > 0$:

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{F}(t, x(t), y_k(t)) \\ \dot{y}_i(t) & = & -q y_i(t) + y_{i-1}(t), \quad i \in \{2, \ldots, k\} \\ \dot{y}_1(t) & = & -q y_1(t) + v(t). \end{array} \right. \tag{2}$$

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We can also allow $\mathcal{F}(t, x, z)$, and actuator errors added to u.

Converging-Input-Converging-State Assumption

Assumption 2: There are a locally Lipschitz bounded ω, and constants $q > 0$, $\tau > 0$, and $T \geq 0$, such that for all continuous δ's that exponentially converge to zero, all solutions $x(t)$ of

$$\dot{x}(t) = \mathcal{F}\left(t, x(t), \mathcal{G}(x_t) + \delta(t)\right), \text{ where} \tag{CICS}$$

$$\mathcal{G}(x_t) = \int_{t-\tau}^{t} \int_{m_{k-1}-\tau}^{m_{k-1}} \cdots \int_{m_1-\tau}^{m_1} e^{q(m_0-t)} \omega(x(m_0 - T)) \mathrm{d}m_0 \ldots \mathrm{d}m_{k-1}$$

satisfy $\lim_{t\to\infty} x(t) = 0$. Also, $\omega(0) = 0$.

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Ex: $\dot{x}(t) = Ax(t) + \delta(t)$ with A Hurwitz. Many nonlinear cases.

Our Theorem

Under Assumptions 1-2, all solutions of

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in closed loop with the bounded control

$$v(t) = \sum_{j=0}^{k} \frac{k!(-1)^j e^{-jq\tau}}{j!(k-j)!} \omega\big(x(t - j\tau - T)\big), \tag{4}$$

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Extensions: If $\dot{x}(t) = \mathcal{F}\,(t, x(t), \mathcal{G}(x_t) + \delta(t))$ is ISS, then we can prove ISS of (3) with respect to additive uncertainty on v.

Ex: Khalil's 2002 Nonlinear Systems, 3rd Edition

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$$u_K(x,z) = -\frac{\partial V_0}{\partial z_1}(x,z_1) + \frac{\partial \phi}{\partial z_1}(x,z_1)z_2 - z_2$$
$$+\frac{\partial \phi}{\partial x}(x,z_1)(x^2 - x^3 + z_1) + \phi(x,z_1),$$

where $V_0(x,z_1) = \frac{1}{2}x^2 + \frac{1}{2}(z_1 + x + x^2)^2$ and

$$\phi(x,z_1) = -2x - (1+2x)(x^2 - x^3 + z_1) - z_1 - x^2,$$

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$$u_J(t) = \frac{q^2}{(1-e^{-q\tau})^2}\{\omega(x(t-T)) - 2e^{-q\tau}\omega(x(t-\tau-T))$$
$$+e^{-2q\tau}\omega(x(t-2\tau-T))\} - 2qz_2(t) - q^2 z_1(t),$$

where $\omega(x) = -\sin\left(\frac{\pi x}{2}\right)\mathbf{1}_{[-2,2]}(x)$, $T = .055$, $q = 1/\tau$, $\tau = .001$.

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Problem: Given a function $\mathcal{F}$ that satisfies Assumption 1, find the largest T such that: There exist a function ω and constants $q > 0$ and $\tau > 0$ such that Assumption 2 is satisfied.

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Thank you for your attention!

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Nonlinear control design via Lyapunov functions and positivity-based techniques

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Abstract

We will present fundamental results pertaining to ordinary differential equations, discrete-time systems and nonlinear control theory. In particular, we will review the notion of Lyapunov function, the LaSalle Invariance Principle, the Jurdjevic-Quinn's theorem and the techniques called backstepping and forwarding. We will perform construction of strict Lyapunov functions. We will study the notion of positive systems. We will study several applied problems (chemostats, PVTOL, cart-pendulum system).

The module is partially based on the research monograph: M. Malisoff, F. Mazenc, Constructions of Strict Lyapunov Functions, Spinger-Verlag, serie : Communications and Control Engineering, 2009

Outline :

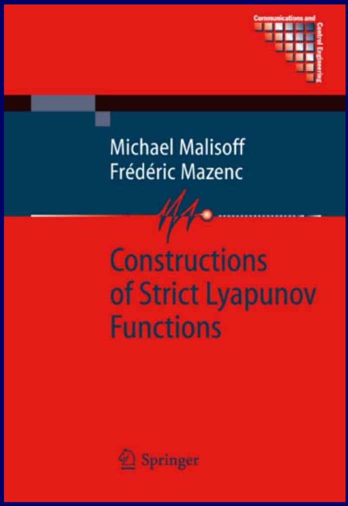
1) **Introduction to dynamical systems:** Ordinary Differential Equations, discrete-time systems, time-varying systems, basic notions (existence and uniqueness of solutions, finite escape time phenomenon). Notions of stability (local, global, basin of attraction), notion of input-to-state stability.

2) **Fundamental results. Linear systems:** stability analysis, linearization. Hartman-Grobman Theorem, Two dimentional systems : Poincaré–Bendixson theorem. Dulac's criterion, properties of ω-limit sets.

3) **Lyapunov functions:** Lyapunov theorem, converse Lyapunov theorem, LaSalle Invariance Principle. Weak Lyapunov functions, strict Lyapunov functions, Matrosov Theorem. Construction of strict Lyapunov functions. Determination of an estimate of a basin of attraction via a strict Lyapunov functions. Notion of ISS Lyapunov function.

4) **Control design:** Lyapunov design, Jurdjevic-Quinn theorem, classical backstepping, bounded backstepping, backstepping for time-varying systems, stabilization and tracking though forwarding, Sontag's formula.

5) **Positive systems:** Cooperative nonlinear systems, linear positive systems, linear Lyapunov function. Notion of interval observer.



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