New Prediction Approach for Stabilizing Time-Varying Systems under Time-Varying Input Delay

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Background and Motivation

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Z. Artstein, I. Karafyllis, M. Krstic, S. Niculescu, P. Pepe, ...

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Assumption 1: The functions A and B are bounded and continuous, and there is a known bounded continuous function $K : [0, \infty) \to \mathbb{R}^{\ell \times n}$ such that $\dot{x}(t) = [A(t) + B(t)K(t)]x(t)$ is uniformly globally exponentially stable to 0.

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Assumption 2: The function $h : \mathbb{R} \to [0, \infty)$ is C^1 and bounded from above by a constant $c_h > 0$. Also, its derivative $\dot{h}$ is bounded from below, and $\dot{h}$ is bounded from above by a constant $l_h \in (0, 1)$, and $\dot{h}$ has a global Lipschitz constant $n_h > 0$.

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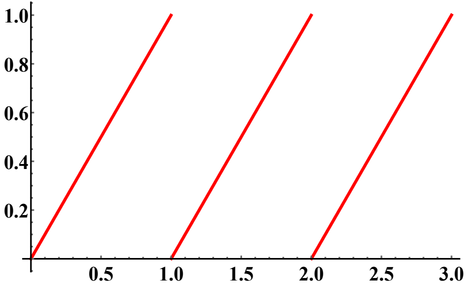

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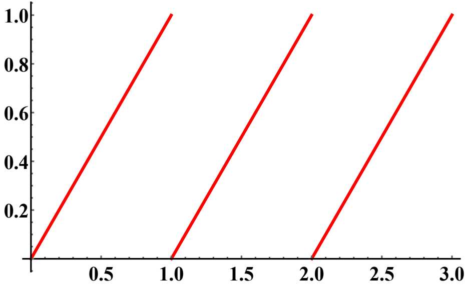

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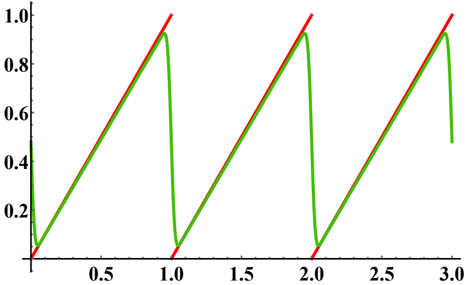

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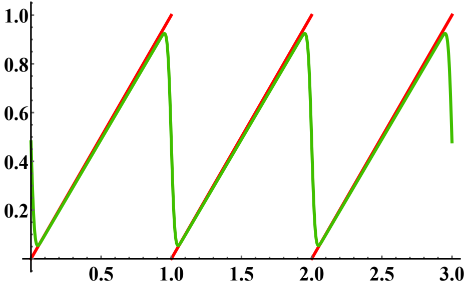

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Assumption 2 holds with $c_h = 0.924$, $l_h = 0.98$, and $n_h = 592.72$.

Preliminaries for Our Theorem

We use an mn-dimensional dynamic extension to build our delay compensating control for any

$$m > \max\left\{2, 4\left(\frac{b_1}{\sqrt{2}} + b_2\right)\frac{c_h}{1-l_h}\right\}, \tag{LB}$$

where

$$b_1 = \left[1 + \left(1 + \frac{u_c}{m}\right)^m |A|_\infty\right]\left(1 + \frac{u_c}{m}\right)^m |A|_\infty,$$

$$b_2 = \left[1 + \left(1 + \frac{u_c}{m}\right)^m |A|_\infty\right]^2\left(1 + \frac{u_c}{m}\right), \quad \text{and} \quad u_c = \frac{c_h n_h}{(1-l_h)^2} + \frac{l_h}{1-l_h}.$$

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Notation: $\Omega_i(t) = t - \frac{i}{m}h(t)$ and $\theta_i(t) = \Omega^{-1}_{m-i+1}(\Omega_{m-i}(t))$ for $i \in \{0, ..., m\}$, $R_1(t) = \dot{\theta}_1(t)$, $R_i(t) = \dot{\theta}_i(t)R_{i-1}(\theta_i(t))$ for $i > 1$.

Our Theorem

Let Assumptions 1-2 hold and m satisfy (LB). Then if we use the control $u(t) = K(\Omega_m^{-1}(t))z_m(t)$ in (LTV), where z_m is the last n components of the system

$$
\begin{array}{rcl}
\dot{z}_1(t) & = & R_1(t)A(\theta_1(t))z_1(t) + R_1(t)B(\theta_1(t))u(\Omega_{m-1}(t)) \\
 & & + L_1(t)[z_1(\theta_1^{-1}(t)) - x(t)] \\
\dot{z}_i(t) & = & R_i(t)A(G_i(t))z_i(t) + R_i(t)B(G_i(t))u(\Omega_{m-i}(t)) \\
 & & + L_i(t)[z_i(\theta_i^{-1}(t)) - z_{i-1}(t)],\ i \in \{2, \dots, m\}
\end{array}
\tag{1}
$$

where $L_i(t) = -I_n - R_i(t)A(G_i(t))$ and $G_i = \Omega_m^{-1} \circ \Omega_{m-i}$, then the dynamics for $(x, \mathcal{E})$ are globally exponentially stable to 0, where $\mathcal{E}(t) = (z_1(t) - x(\theta_1(t)), z_2(t) - z_1(\theta_2(t)), \dots, z_m(t) - z_{m-1}(\theta_m(t)))$.

Pendulum Example

$$\left\{\begin{array}{rcl}\dot{r}_1(t) & = & r_2(t)\\ \dot{r}_2(t) & = & -\frac{g}{l}\sin(r_1(t)) + \frac{1}{\mathcal{M}l^2}v(t-h(t))\end{array}\right. \tag{2}$$

Change of feedback and linearizing the tracking dynamics for tracking $(\omega t, \omega)$ for any $\omega > 0$ gives

$$\left\{\begin{array}{rcl}\dot{x}_1(t) & = & x_2(t)\\ \dot{x}_2(t) & = & -\frac{g}{l}\cos(\omega t)x_1(t) + u(t-h(t))\end{array}\right. \tag{3}$$

Theorem applies with $h(t) = 1 + \alpha\sin(t)$ with $\alpha \in (0,1)$.

E.g., if $l > g$ and $\omega > 0$ and $\alpha = 1/7$, can pick $m = 47$.

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We hope to prove generalizations for ODE-PDE cascades.