Robustness of Nonlinear Systems with Respect to Delay and Sampling of the Controls

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Statement of Problem

We are given a UGAS nonlinear time varying system

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t)) u(t, x(t)) \quad (CL)$$

satisfying assumptions we list later.

For all \( t \in \mathbb{R} \),

$$u(t, 0) = 0$$

Given a constant \( \nu > 0 \), we want to compute upper bounds on \( \delta + \tau \) such that the input delayed sampled system defined by

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t)) u(t_i - \tau, x(t_i - \tau))$$

is UGAS for all sampling points satisfying \( t_i + 1 - t_i \in [\nu, \delta] \) for all \( i \).
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\[ \dot{x}(t) = f(t, x(t)) + g(t, x(t))u_s(t, x(t)) \]  \hspace{1cm} (CL)

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Sampling and Delays: Fridman, Jiang, Karafyllis-Krstic, Mirkin,…
Our Two Assumptions

Assumption 1: There are a positive definite radially unbounded function $V$ and a continuous positive definite function $W$ such that $\dot{V}(t,x) \leq -W(x)$ along all trajectories of (CL), $u$ is $C^1$, and $f$ and $g$ are locally Lipschitz in $x$ and piecewise continuous in $t$.

Assumption 2: There are constants $c_i > 0$ such that
\[
\left| \frac{\partial u_s}{\partial x}(t,x) \right| g(t,x) \leq c_3 W(x),
\]
\[
\left| \frac{\partial V}{\partial x}(t,x) \right| g(t,x) \leq c_2 W(x),
\]
\[
\left| \frac{\partial u_s}{\partial x}(t,x) \right| g(t,x) \leq c_4 \left[ V(t,x) + 1 \right]
\]
hold for all $t \geq 0$ and $x \in \mathbb{R}^n$. Here $\dot{u}_s(t,x) = \frac{\partial u_s}{\partial t}(t,x) + \frac{\partial u_s}{\partial x}(t,x) \left( f(t,x) + g(t,x) u_s(t,x) \right)$.
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**Assumption 1**: There are a $C^1$ positive definite radially unbounded function $V$ and a continuous positive definite function $W$ such that $\dot{V}(t, x) \leq -W(x)$ along all trajectories of (CL), $u_s$ is $C^1$, and $f$ and $g$ are locally Lipschitz in $x$ and piecewise continuous in $t$. 
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Assumption 2: There are constants $c_i > 0$ such that

\[
\left| \frac{\partial u_s}{\partial x}(t, x) g(t, x) \right|^2 \leq c_1, \quad \left| \frac{\partial V}{\partial x}(t, x) g(t, x) \right|^2 \leq c_2 W(x),
\]

(1)

\[
\left| u_s(t, x) \right|^2 \leq c_3 W(x), \quad \text{and}
\]

(2)

\[
\left| \frac{\partial V}{\partial x}(t, x) g(t, x) u_s(t, x) \right| \leq c_4 [V(t, x) + 1]
\]

(3)

hold for all $t \geq 0$ and $x \in \mathbb{R}^n$. 
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$$\left| u_s(t, x) \right|^2 \leq c_3 W(x), \text{ and}$$

$$\left| \frac{\partial V}{\partial x}(t, x)g(t, x)u_s(t, x) \right| \leq c_4 [V(t, x) + 1]$$

hold for all $t \geq 0$ and $x \in \mathbb{R}^n$.

Here $\dot{u}_s(t, x) = \frac{\partial u_s}{\partial t}(t, x) + \frac{\partial u_s}{\partial x}(t, x)(f(t, x) + g(t, x)u_s(t, x))$. 
Main Result (M-Mazenc-Dinh, *Automatica*, June 2013)

**Theorem:** Let Assumptions 1-2 hold. If $\delta$ and $\tau^*$ are any two positive constants such that

$$\delta + \tau^* \leq \sqrt{\frac{4}{c_1} + 8c_2c_3}$$

and if $\tau \in (0, \tau^*)$, then the system

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t))u(t_i - \tau, x(t_i - \tau)),$$

for any sequence $\{t_i\}$ satisfying $t_i + 1 - t_i \in [\nu, \delta]$ for all $i$ is UGAS.

**Remarks:** Covers dynamics that are not necessarily globally Lipschitz or locally exponentially stable. The proof is based on a functional of a new type.
Theorem: Let Assumptions 1-2 hold. If $\delta$ and $\tau_*$ are any two positive constants such that

$$\delta + \tau_* \leq \frac{1}{\sqrt{4c_1 + 8c_2c_3}}$$

and if $\tau \in (0, \tau_*)$, then the system

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t))u_s(t_i - \tau, x(t_i - \tau)), \quad t_i \leq t < t_{i+1}$$

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Remarks: Covers dynamics that are not necessarily globally Lipschitz or locally exponentially stable. The proof is based on a functional of a new type.
Sketch of Proof

Choose $U(t,x_t) = V(t,x(t)) + \Gamma(t,x_t)$, where $V$ is the Lyapunov function for the undelayed nonsampled system as before, and $\Gamma(t,x_t) = \frac{1}{4} c_3 (\delta + \tau^*) \int_{t-\delta-\tau^*}^{t} |\psi(m,x_m)|^2 dm \int_{\ell}$, for $t \in [t_i, t_i+1)$ and all $i$ as before.

Then $\dot{U} \leq -\frac{1}{4} W(x(t)) - \frac{1}{8} c_3 (\delta + \tau^*) \int_{t-\delta-\tau^*}^{t} |\psi(m,x_m)|^2 dm$ along (SD).

We can find $C_1 K_\infty$ functions $\kappa$ and $\gamma$ such that $U_{\kappa} = \kappa(U)$ satisfies $\dot{U_{\kappa}} \leq -\gamma(U_{\kappa})$ along all trajectories of (SD).
Choose $U(t, x_t) = V(t, x(t)) + \Gamma(t, x_t)$, where $V$ is the Lyapunov function for the undelayed nonsampled system as before,

$$\Gamma(t, x_t) = \frac{1}{4c_3(\delta+\tau^*)} \int_t^t \left[ \int_{t-\delta-\tau^*}^t |\psi(m, x_m)|^2 \, dm \right] \, d\ell,$$

$$\psi(t, x_t) = \frac{\partial u_s}{\partial t}(t, x(t)) + \frac{\partial u_s}{\partial x}(t, x(t)) \dot{x}(t),$$

and

$$\dot{x}(t) = f(t, x(t)) + g(t, x(t)) u_s(t_i - \tau, x(t_i - \tau))$$ \hspace{1cm} (SD)

for $t \in [t_i, t_{i+1})$ and all $i$ as before.
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$$\dot{x}(t) = f(t, x(t)) + g(t, x(t)) u_s(t_i - \tau, x(t_i - \tau)) \quad \text{(SD)}$$

for $t \in [t_i, t_{i+1})$ and all $i$ as before. Then

$$\dot{U} \leq -\frac{1}{4} W(x(t)) - \frac{1}{8c_3(\delta+\tau_*)} \int_{t-\delta-\tau_*}^{t} |\psi(m, x_m)|^2 \, dm$$

along (SD).
Sketch of Proof

Choose \( U(t, x_t) = V(t, x(t)) + \Gamma(t, x_t) \), where \( V \) is the Lyapunov function for the undelayed nonsampled system as before,

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\psi(t, x_t) = \frac{\partial u_s}{\partial t}(t, x(t)) + \frac{\partial u_s}{\partial x}(t, x(t)) \dot{x}(t), \quad \text{and}
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\dot{x}(t) = f(t, x(t)) + g(t, x(t)) u_s(t_i - \tau, x(t_i - \tau)) \quad \text{(SD)}
\]

for \( t \in [t_i, t_{i+1}) \) and all \( i \) as before. Then

\[
\dot{U} \leq -\frac{1}{4} \mathcal{W}(x(t)) - \frac{1}{8c_3(\delta + \delta_*)} \int_{t-\delta-\delta_*}^{t} |\psi(m, x_m)|^2 \, dm
\]

along (SD). We can find \( C^1 \mathcal{K}_\infty \) functions \( \kappa \) and \( \gamma \) such that \( U_\kappa = \kappa(U) \) satisfies \( \dot{U}_\kappa \leq -\gamma(U_\kappa) \) along all trajectories of (SD).
As special cases, our theorem covers a large class of nonsampled delayed examples that were beyond the scope of [MML08].

Proposition: If a system \( \dot{x} = f(x) + u \) with \( f \) bounded is rendered GAS on \( \mathbb{R}^n \) by a bounded feedback \( u(s(x)) \), then for each Lyapunov function \( V(t, x) \) of the closed loop system, the requirements from [MML08] on the delayed system

\[ \dot{x}(t) = f(x(t)) + u(s(x(t - \tau))) \quad (5) \]

fail to hold.
Special Case of Delays but No Sampling

MML08 Mazenc, F., M. Malisoff, and Z. Lin, ”Further results on input-to-state stability for nonlinear systems with delayed feedbacks,” Automatica, 44(9), pp. 2415-2421, 2008.
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$$\dot{x}(t) = f(x(t)) + u_s(x(t - \tau))$$

fail to hold.
Nonlinear Examples

We do not require global Lipschitzness of the dynamics or exponential stabilizability.

Example 1:
\[ \dot{x} = x^2 + 1 + x^2 u \quad (6) \]
\[ u_s(x) = -x \quad \text{and} \quad V(x) = \frac{1}{2} x^2. \]

Example 2:
\[ \dot{x}_1 = -x_1 - x_9 + x_2 \]
\[ \dot{x}_2 = u + x_1 \quad (7) \]
\[ u_s(t, x) = -x_1 - x_2, \quad \text{and} \quad V(t, x) = 0. \]
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\( n = 2 \) example:
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\begin{align*}
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\dot{x}_2 &= u + x_1
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$n = 2$ example:

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\dot{x}_2 &= u + x_1
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\]

\[ u_s(t, x) = -x_1 - x_2, \quad \text{and} \quad V(t, x) = 0.1x_1^{10} + 0.5x_1^2 + 2x_2^2. \]
Tracking Example (Wheeled Robot Kinematics...)

\[
\dot{x}_1 = \omega x_2 \\
\dot{x}_2 = -\omega x_1 + \lambda \dot{x}_3 \\
\dot{x}_3 = \omega,
\]

where \(\lambda\) and \(\omega\) are controls.

We wish to track \((0, 0, -\cos(\zeta t))\).

The change of feedback \(\omega = \zeta \sin(\zeta t) + \mu\) produces

\[
\dot{x}_1 = (\zeta \sin(\zeta t) + \mu)x_2 \\
\dot{x}_2 = -\left(\zeta \sin(\zeta t) + \mu\right)x_1 + \lambda \dot{x}_3 \\
\dot{x}_3 = \mu.
\]

\[\mu(z(t_i - \tau)) = -\zeta \mu z(t_i - \tau) \sqrt{1 + z^2(t_i - \tau)}, \quad \lambda(x(t_i - \tau)) = -\zeta x(t_i - \tau) \delta + \tau^* < \frac{1}{2} \zeta \min\{\sqrt{\zeta}, \frac{1}{53}(\zeta + 1)\}\]
Jiang, Lefeber, Nijmeijer, SCL’01

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\]

(9)

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\mu(z(t_i - \tau)) = -\zeta \frac{\delta z(t_i - \tau)}{\sqrt{1 + z^2(t_i - \tau)}}, \quad \lambda(x(t_i - \tau)) = -\zeta x_2(t_i - \tau)
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\dot{z} &= \mu.
\end{align*}
\]

(9)

\[
\mu(z(ti - \tau)) = -\zeta \frac{d_{\delta}z(ti - \tau)}{\sqrt{1 + z^2(ti - \tau)}}, \quad \lambda(x(ti - \tau)) = -\zeta x_2(ti - \tau)
\]

\[
\delta + \tau_* < \frac{1}{2\zeta} \min \left\{ \frac{1}{\sqrt{3\delta}}, \frac{1}{53(\delta+1)} \right\}
\]

(10)
Conclusions

Delays and sampling in controls often occur together but robustness to delays and sampling has seldom been studied. Introducing even arbitrarily small input delays into a uniformly globally stabilizing controller can lead to instability. Using a new kind of functional, we found direct formulas for delay and sampling bounds that maintain UGAS without an offset. Our work allows perturbed sampling and saturating controls, and does not require local exponential stability or global Lipschitzness. We illustrated our work in an interesting dynamics from the kinematics of wheeled mobile robots. We hope to extend our work to time varying or state dependent delays, and systems that are not control affine.
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