Adaptive Control with Parameter Identification with an Application to Curve Tracking

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Sponsor: NSF Energy, Power, and Adaptive Systems
Joint with Fumin Zhang from Georgia Tech ECE

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Adaptive Tracking and Parameter Identification

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Specify u to get a *doubly* parameterized closed loop family

$$Y'(t) = \mathcal{G}(t, Y(t), Y(t-\tau), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}, \tag{2}$$

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Problem: Given a desired reference trajectory Y_r, specify u and a dynamics for an estimate $\hat{\Gamma}$ of Γ such that the augmented error $\mathcal{E}(t) = (Y(t) - Y_r(t), \Gamma - \hat{\Gamma}(t))$ satisfies ISS with respect to δ.

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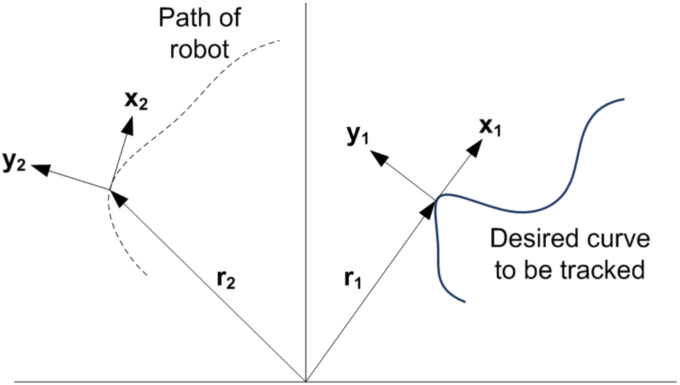

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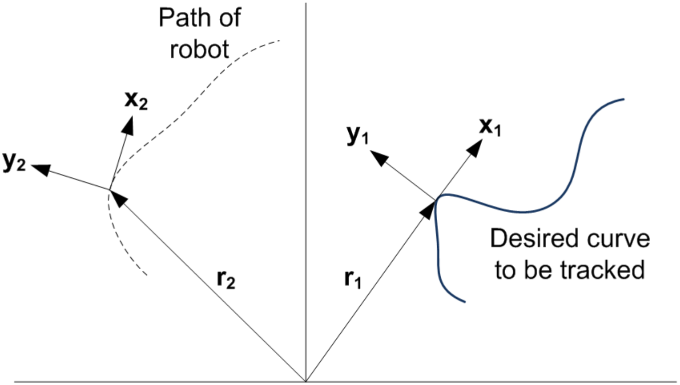

$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

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Feedback linearization with $z = \sin(\phi)$ cannot be applied.

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This gives UGAS, using LaSalle Invariance.

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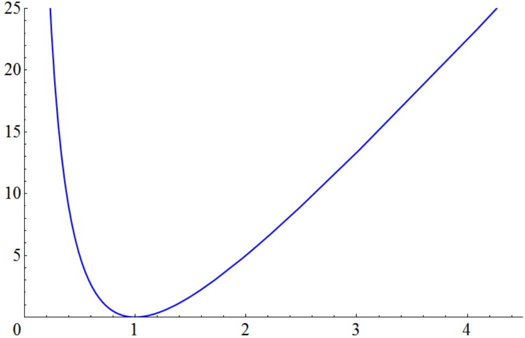
To realize our goals, we added assumptions on *h* which hold for



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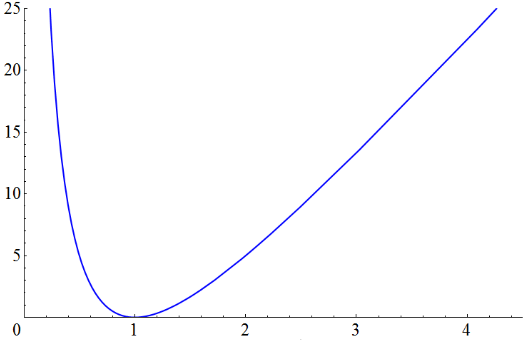

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See my Automatica and TAC papers with Fumin Zhang.

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Estimator: $$\dot{\widehat{\Gamma}} = (\widehat{\Gamma} - c_{\min})(c_{\max} - \widehat{\Gamma}) \frac{\partial V^{\sharp}(\rho, \phi)}{\partial \phi} u(\rho, \phi, \hat{\Gamma}) \quad (7)$$

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$$V^\sharp(\rho,\phi) = -h'(\rho)\sin(\phi) + \int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m \qquad (8)$$

$$\gamma(q) = \frac{1}{\mu}\left(\frac{2}{\alpha^2\rho_0^4}(q+2\alpha\rho_0)^3 + 1\right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4\alpha^2}q^3 \qquad (9)$$

$$V(\rho,\phi) = -\ln\left(\cos(\phi)\right) + h(\rho) \qquad (10)$$

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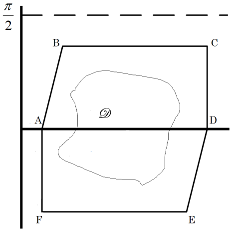

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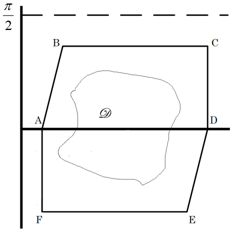

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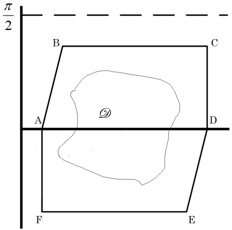

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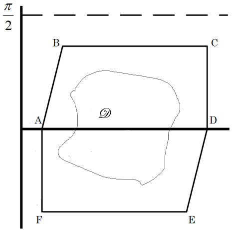
For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Robustly Forwardly Invariant Hexagonal Regions

Restrict the perturbations $\delta(t)$ to keep the state $X = (\rho, \phi)$ from leaving the state space $\mathcal{X} = (0, \infty) \times (-\pi/2, \pi/2)$.



View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a nested union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$. [For each i, all trajectories of (Σ_c) starting in H_i for all $\delta : [0, \infty) \to [-\delta_{*i}, \delta_{*i}]$ stay in H_i.] The tilted legs have slope $c_{\min}\mu/c_{\max}$.

For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Then we prove ISS of the tracking and parameter identification system on each set H_i, with the disturbance set $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

References with Hyperlinks

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In our future work, we will study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.