Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

Michael Malisoff
Adaptive Control Problems: Basic Framework

\[ Y'(t) = \mathcal{F}(t, Y(t), u(t, \hat{\Gamma}(t), Y(t)), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}. \quad (1) \]

\( \mathcal{Y} \subseteq \mathbb{R}^n. \) \( \delta : [0, \infty) \rightarrow \mathcal{D} \) is (nonstochastic) uncertainty. \( \mathcal{D} \subseteq \mathbb{R}^m. \)

The vector \( \Gamma \) is constant but unknown. \( u \) is a control.
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The control \( u \) and \( \hat{\Gamma}'(t) = \mathcal{H}(t, \hat{\Gamma}(t), Y(t), u(t, \hat{\Gamma}(t), Y(t))) \) will be chosen so that each solution \( Y : [t_0, t_{\text{max}}) \rightarrow \mathcal{Y} \) of (2) for each initial state \( Y(t_0) \in \mathcal{Y} \) and each \( \delta \) is uniquely defined in \( [t_0, \infty) \).
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Problem: Given \( Y_R : [0, \infty) \rightarrow \mathcal{Y} \), find \( u \) and a dynamics for an estimate \( \hat{\Gamma} \) of \( \Gamma \) such that the dynamics for the augmented error \( \mathcal{E}(t) = (Y(t) - Y_R(t), \Gamma - \hat{\Gamma}(t)) \) satisfies ISS with respect to \( \delta \).
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**Persistent excitation.** Required nondegeneracy condition on \( Y_R \).
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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...
Adaptive Control Problems: Basic Framework

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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...
Input-to-State Stability or ISS (Sontag, ’89)

Definition: A system \( \dot{E}(t) = G(t, E(t), \Gamma) \) is uniformly globally asymptotically stable to 0 provided there are \( \gamma_1 \) and \( \gamma_2 \) in \( K_\infty \) such that for all of its solutions \( E : [t_0, t_{\text{max}}) \rightarrow S \), we have

\[
|E(t)| \leq \gamma_1(e^{t_0-t}\gamma_2(|E(t_0)|)) \quad \text{for all} \quad t \geq t_0.
\]
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$\gamma_i \in \mathcal{K}_\infty$: $\gamma_i$’s continuous, 0 at 0, strictly increasing, unbounded.
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Prove ISS by building certain strict Lyapunov functions.
Summary of Some of My Teams’ Research

For many systems, we design controls $u(t, \Gamma, Y(t))$ that ensure ISS under uncertainties $\delta$.

Interconnect the systems with dynamics for estimators $\hat{\Gamma}(t)$ that converge to $\Gamma$ from all $\hat{\Gamma}(0)$'s, and then replace $\Gamma$ in $u$ by $\hat{\Gamma}$.

For state space subsets $Y^{\flat} \subseteq Y$, compute maximal perturbation sets $D^{\flat} \subseteq D$ that ensure strong forward invariance of $Y^{\flat}$.

Bioreactors, DC motors, general theory, heart rate controllers, helicopters, human-computer interactions, magnetic bearings, marine robots, microelectromechanical relays, neuromuscular electrical stimulation, unmanned air vehicles,..
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For state space subsets $\mathcal{Y}^b \subseteq \mathcal{Y}$, compute maximal perturbation sets $\mathcal{D}^b \subseteq \mathcal{D}$ that ensure strong forward invariance of $\mathcal{Y}^b$. 
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Sample Theorem (M-Mazenc-de Queiroz)

We solved the tracking and parameter identification problem for

\[
\begin{align*}
\dot{x} &= f(\xi) \\
\dot{z}_i &= g_i(\xi) + k_i(\xi)\theta_i + \psi_i u_i, \quad i = 1, 2, \ldots, s.
\end{align*}
\]

(3)

\[\xi = (x, z) \in \mathbb{R}^{r+s}.\]
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\[\xi = (x, z) \in \mathbb{R}^{r+s}. \quad (\theta, \psi) = (\theta_1, \ldots, \theta_s, \psi_1, \ldots, \psi_s) \in \mathbb{R}^{p_1+\ldots+p_s+s}.\]
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The \(C^2\) reference trajectory \(\xi_R = (x_R, z_R)\) is assumed to have some period \(T > 0\) and satisfy \(\dot{x}_R(t) = f(\xi_R(t))\) for all \(t \geq 0\).
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Main PE Assumption: positive definiteness of the matrices

\[
\mathcal{M}_i = \int_0^T \lambda_i^\top(t)\lambda_i(t) \, dt \in \mathbb{R}^{(p_i+1)\times(p_i+1)}, \quad 1 \leq i \leq s,
\]

(4)

where \(\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))\) for \(i = 1, 2, \ldots, s.\)
Two Other Key Assumptions

A1 We know $v$ and $a$ are continuous on $\mathbb{R}_{r+s} \rightarrow [0, \infty)$ for
\[
\dot{X} = f(X, Z, \xi) - f(\xi) \\
\dot{Z} = v f(t, X, Z)
\] (5)
such that $-\dot{V}$ and $V$ have a lower bound $\bar{c} \mid (X, Z)$ near 0 (with $\bar{c} > 0$ constant), and $V$ and $v f$ have period $T$ in $t$.

Key: Reduces the LF construction problem to (5).

A2 There are known positive constants $\theta_M, \psi$ and $\psi_i$ such that $\psi < \psi_i < \psi$ and $|\theta_i| < \theta_M$ (6) for each $i \in \{1, 2, ..., s\}$.

Known directions for the $\psi_i$'s.
Two Other Key Assumptions

A1 We know $v_f$ and a $C^1$ LF $V : [0, \infty) \times \mathbb{R}^{r+s} \to [0, \infty)$ for

\[
\begin{align*}
\dot{X} & = f((X, Z) + \xi_R(t)) - f(\xi_R(t)) \\
\dot{Z} & = v_f(t, X, Z)
\end{align*}
\]

such that $-\dot{V}$ and $V$ have a lower bound $\bar{c}|(X, Z)|^2$ near 0 (with $\bar{c} > 0$ constant), and $V$ and $v_f$ have period $T$ in $t$.

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Two Other Key Assumptions

A1 We know \( \nu_f \) and a \( C^1 \) LF \( V : [0, \infty) \times \mathbb{R}^{r+s} \rightarrow [0, \infty) \) for

\[
\begin{aligned}
\dot{X} &= f((X, Z) + \xi_R(t)) - f(\xi_R(t)) \\
\dot{Z} &= \nu_f(t, X, Z)
\end{aligned}
\]  

such that \(-\dot{V}\) and \(V\) have a lower bound \( \bar{c}(X, Z)^2 \) near 0 (with \( \bar{c} > 0 \) constant), and \(V\) and \(\nu_f\) have period \(T\) in \(t\).

Key: Reduces the LF construction problem to (5).

A2 There are known positive constants \(\theta_M, \underline{\psi}, \overline{\psi}\) such that

\[
\underline{\psi} < \psi_i < \overline{\psi} \quad \text{and} \quad |\theta_i| < \theta_M
\]

for each \(i \in \{1, 2, \ldots, s\}\). Known directions for the \(\psi_i\)’s.
Dynamic Feedback

The estimator has state space $\hat{S} = \{\prod_{i=1}^{s}(-\theta_M, \theta_M)^{p_i}\} \times (\underline{\psi}, \overline{\psi})^s$:

$$
\begin{align*}
\dot{\hat{\theta}}_{i,j} &= (\hat{\theta}_{i,j}^2 - \theta_M^2) \varpi_{i,j}(t, \tilde{\xi}), \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\dot{\hat{\psi}}_i &= (\hat{\psi}_i - \underline{\psi}) (\hat{\psi}_i - \overline{\psi}) \Upsilon_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}), \quad 1 \leq i \leq s
\end{align*}
$$

(7)

Here $\hat{\theta}_i = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_i})$ for $i = 1, 2, \ldots, s$, $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R$, 

$$
\varpi_{i,j}(t, \tilde{\xi}) = -\frac{\partial V}{\partial \tilde{z}_i}(t, \tilde{\xi}) k_{i,j}(\tilde{\xi} + \xi_R(t))
$$

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\[
\varpi_{i,j}(t, \tilde{\xi}) = -\frac{\partial V}{\partial \tilde{z}_i}(t, \tilde{\xi}) k_{i,j}(\tilde{\xi} + \xi_R(t)) \quad \text{and}
\]

\[
\Upsilon_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}) = -\frac{\partial V}{\partial \tilde{z}_i}(t, \tilde{\xi}) u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}).
\]

(8)

\[
u_{f,i}(t, \tilde{\xi}) = \frac{v_{f,i}(t, \tilde{\xi}) - g_i(\xi) - k_i(\xi) \hat{\theta}_i + \hat{\zeta}_{R,i}(t)}{\hat{\psi}_i}
\]

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\end{align*}$$

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Here $\hat{\theta}_i = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_i})$ for $i = 1, 2, \ldots, s$ , $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R$,

$$\begin{align*}
\varpi_{i,j}(t, \tilde{\xi}) &= -\frac{\partial V}{\partial \tilde{z}_i}(t, \tilde{\xi}) k_{i,j}(\tilde{\xi} + \xi_R(t)) \quad \text{and} \\
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\end{align*}$$

(8)

$$u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}) = \frac{v_{f,i}(t, \tilde{\xi}) - g_i(\xi) - k_i(\xi) \hat{\theta}_i + \dot{z}_{R,i}(t)}{\hat{\psi}_i}$$

(9)

Barrier terms ensure $\underline{\psi} < \hat{\psi}_i(t) < \overline{\psi}$ and $|\hat{\theta}_{i,j}(t)| < \theta_M$ for all $t \geq 0$
Augmented Error Dynamics

Tracking error: $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R)$

Estimation errors: $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$. 
Augmented Error Dynamics

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Augmented Error Dynamics

Tracking error: \( \tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R) \)

Estimation errors: \( \tilde{\theta}_i = \theta_i - \hat{\theta}_i \) and \( \tilde{\psi}_i = \psi_i - \hat{\psi}_i \). \( \mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) \).

\[
\begin{align*}
\dot{\tilde{x}} &= f(\tilde{\xi} + \xi_R(t)) - f(\xi_R(t)) \\
\dot{\tilde{z}}_i &= v_{f,i}(t, \tilde{\xi}) + k_i(\tilde{\xi} + \xi_R(t)) \tilde{\theta}_i \\
&\quad + \tilde{\psi}_i u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}), \quad 1 \leq i \leq s \\
\dot{\tilde{\theta}}_{i,j} &= -\left(\hat{\theta}_{i,j}^2 - \theta_M^2\right) \varpi_{i,j}, \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\hat{\psi}_i &= -\left(\hat{\psi}_i - \psi\right) (\hat{\psi}_i - \overline{\psi}) \gamma_i, \quad 1 \leq i \leq s.
\end{align*}
\] (AED)
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Estimation errors: $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$. $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi})$.

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\dot{\tilde{z}}_i &= v_{f,i}(t, \tilde{\xi}) + k_i(\tilde{\xi} + \xi_R(t))\tilde{\theta}_i \\
&\quad + \tilde{\psi}_i u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}), \quad 1 \leq i \leq s \\
\dot{\tilde{\theta}}_{i,j} &= -\left(\hat{\theta}_{i,j}^2 - \theta_M^2\right)\varpi_{i,j}, \quad 1 \leq i \leq s, 1 \leq j \leq p_i \\
\dot{\tilde{\psi}}_i &= -\left(\hat{\psi}_i - \psi\right)\left(\hat{\psi}_i - \bar{\psi}\right)\gamma_i, \quad 1 \leq i \leq s.
\end{align*}
\]

(AED)

\[
S = \mathbb{R}^{r+s} \times \left(\prod_{i=1}^s \left\{\prod_{j=1}^{p_i} (\theta_{i,j} - \theta_M, \theta_{i,j} + \theta_M)\right\}\right) \\
\times \left(\prod_{i=1}^s (\psi_i - \bar{\psi}, \psi_i - \bar{\psi})\right).
\]
Stabilization Analysis

We build a strict LF for the augmented error dynamics for \( E = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( S \).

We start with this nonstrict barrier type LF on \( S \):

\[
V_1(t, E) = V(t, \tilde{\xi}) + s \sum_{i=1}^{p} \int_{0}^{\theta_i,j} m \theta_i^2 M - (m - \theta_i,j)^2 dm + s \sum_{i=1}^{p} \int_{0}^{\psi_i} m(\psi_i - \psi_m)(\psi - \psi_i + \psi_m) dm.
\]

There is a positive definite function \( W \) such that \( \dot{V}_1 \leq -W(\tilde{\xi}) \) along all solutions \( E : [0, \infty) \rightarrow S \) of (AED). This allows \( \dot{V}_1 = 0 \) at some nonzero \( E \)'s, so \( V_1 \) is nonstrict.
Stabilization Analysis

We build a strict LF for the augmented error dynamics for $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $\mathcal{S}$. 
**Stabilization Analysis**

We build a strict LF for the augmented error dynamics for $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $S$.

We start with this nonstrict barrier type LF on $S$:

$$V_1(t, \mathcal{E}) = V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_{0}^{\tilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \, dm$$

$$+ \sum_{i=1}^{s} \int_{0}^{\tilde{\psi}_i} \frac{m}{(\psi_i - m - \psi)(\psi - \psi_i + m)} \, dm.$$

There is a positive definite function $\mathcal{W}$ such that $\dot{V}_1 \leq -\mathcal{W}(\tilde{\xi})$ along all solutions $\mathcal{E} : [0, \infty) \rightarrow S$ of (AED).
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There is a positive definite function $W$ such that $\dot{V}_1 \leq -W(\tilde{\xi})$ along all solutions $\mathcal{E} : [0, \infty) \rightarrow S$ of (AED).

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\]

There is a positive definite function \( W \) such that \( \dot{V}_1 \leq -W(\tilde{\xi}) \) along all solutions \( \mathcal{E} : [0, \infty) \to S \) of (AED).

We transform \( V_1 \) into the desired strict LF \( V^\# \) for (AED).
Stabilization Analysis

We build a strict LF for the augmented error dynamics for
\( E = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi}) \) on its state space \( S \).

We start with this nonstrict barrier type LF on \( S \):

\[
V_1(t, E) = V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\tilde{\theta}_{i,j}} \frac{m}{\theta_{M}^2 - (m - \theta_{i,j})^2} \, dm + \sum_{i=1}^{s} \int_0^{\tilde{\psi}_i} \frac{m}{(\psi_i - m - \psi)(\psi - \psi_i + m)} \, dm.
\]

There is a positive definite function \( W \) such that \( \dot{V}_1 \leq -W(\tilde{\xi}) \) along all solutions \( E : [0, \infty) \rightarrow S \) of (AED).

\( V^\# \) enables proving ISS and rate of convergence analysis.
Our Transformation (M-M-dQ)

Theorem: We can construct a function
$L \in K^\infty \cap C_1$
such that
$V^\#(t, E) = L(V_1(t, E)) + \sum_{i=1}^{\Omega_i(t, E)}$

where
$\Omega_i(t, E) = -\tilde{z}_i^\lambda_i(t) \alpha_i(E) + \frac{1}{T} \psi_i^\top \Omega_i(t) \alpha_i(E)$,
$\alpha_i(E) = [\tilde{\theta}_i^\psi_i - \theta_i \tilde{\psi}_i]$

is a strict LF for (AED) on its state space $S$, so (AED) is UGAS.
Our Transformation (M-M-dQ)

Theorem: We can construct a function $L \in K_\infty \cap C^1$ such that

$$V^\#(t, \mathcal{E}) = L(V_1(t, \mathcal{E})) + \sum_{i=1}^{s} \Omega_i(t, \mathcal{E}), \quad (10)$$

where

$$\Omega_i(t, \mathcal{E}) = -\tilde{Z}_i \lambda_i(t) \alpha_i(\mathcal{E}) + \frac{1}{T\psi} \alpha_i^\top(\mathcal{E}) \Omega_i(t) \alpha_i(\mathcal{E}),$$

$$\alpha_i(\mathcal{E}) = \begin{bmatrix} \tilde{\theta}_i \psi_i - \theta_i \tilde{\psi}_i \\ \tilde{\psi}_i \end{bmatrix}, \quad \Omega_i(t) = \int_{t-T}^{t} \int_{m}^{t} \lambda_i^\top(s) \lambda_i(s) ds \, dm, \quad (11)$$

and $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$

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One Application: Marine Robots

We applied our adaptive approach to curve tracking problems for gyroscopic models for marine robots in a lagoon. We worked with Fumin Zhang’s robotics group in Georgia Tech ECE to search for pollution from the Deepwater Horizon disaster. We combined our adaptive control methods with robust forward invariance to satisfy performance and safety bounds. Robust forward invariance computes maximum allowable disturbance sets $D$ that keep us in state constraint sets $Y$. We combined mathematical analysis with 2 weeks of field work with robotics students at a polluted lagoon at Grand Isle, LA.
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Hyperlinked Related References


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Our Other Adaptive Control Applications

Brushless DC motors turning a mechanical load with uncertain motor electric parameters including integral ISS analysis. Variants for uncertain parameters $\Gamma$ that enter the system in a nonlinear way for curve tracking with unknown curvatures. To also allow delays $\tau$ in state observations in our controls, we convert our strict LF into Lyapunov-Krasovskii functionals. We used artificial neural network expansions for extensions to cases where the $\Gamma$ need not be constant.

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Thanks for your interest!