Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

Michael Malisoff

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Adaptive Control Problems: Basic Framework

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, \hat{\Gamma}(t), Y(t)), \Gamma, \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \tag{1}$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. $\delta : [0, \infty) \to \mathcal{D}$ is (nonstochastic) uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
The vector Γ is constant but unknown. u is a control.

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The control u and $\hat{\Gamma}'(t) = \mathcal{H}(t, \hat{\Gamma}(t), Y(t), u(t, \hat{\Gamma}(t), Y(t)))$ will be chosen so that each solution $Y : [t_0, t_{\max}) \to \mathcal{Y}$ of (2) for each initial state $Y(t_0) \in \mathcal{Y}$ and each δ is uniquely defined in $[t_0, \infty)$.

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Persistent excitation. Required nondegeneracy condition on Y_R.

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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...

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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...

Input-to-State Stability or ISS (Sontag, '89)

Definition: A system $\mathcal{E}'(t) = \mathcal{G}(t, \mathcal{E}(t), \Gamma)$ is uniformly globally asymptotically stable to 0 provided there are γ_1 and γ_2 in $\mathcal{K}_\infty$ such that for all of its solutions $\mathcal{E} : [t_0, t_{\max}) \to \mathcal{S}$, we have $|\mathcal{E}(t)| \le \gamma_1(e^{t_0 - t}\gamma_2(|\mathcal{E}(t_0)|))$ for all $t \ge t_0$.

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Prove ISS by building certain strict Lyapunov functions.

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Sample Theorem (M-Mazenc-de Queiroz)

We solved the tracking and parameter identification problem for

$$\left\{ \begin{array}{rcl} \dot{x} & = & f(\xi) \\ \dot{z}_i & = & g_i(\xi) + k_i(\xi)\theta_i + \psi_i u_i, \quad i = 1, 2, \ldots, s\ . \end{array} \right. \tag{3}$$

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Main PE Assumption: positive definiteness of the matrices

$$\mathcal{M}_i = \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t \;\in\; \mathbb{R}^{(p_i+1)\times(p_i+1)},\; 1 \leq i \leq s, \tag{4}$$

where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for $i = 1, 2, \ldots, s$.

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such that $-\dot{V}$ and V have a lower bound $\bar{c}|(X,Z)|^2$ near 0 (with $\bar{c} > 0$ constant), and V and v_f have period T in t.

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A2 There are known positive constants θ_M, $\underline{\psi}$ and $\overline{\psi}$ such that

$$\underline{\psi} < \psi_i < \overline{\psi} \quad \text{and} \quad |\theta_i| < \theta_M \tag{6}$$

for each $i \in \{1, 2, \ldots, s\}$. Known directions for the ψ_i's.

Dynamic Feedback

The estimator has state space $\hat{\mathcal{S}} = \{\prod_{i=1}^{s}(-\theta_M, \theta_M)^{p_i}\} \times (\underline{\psi}, \overline{\psi})^s$:

$$\left\{ \begin{array}{rcl} \dot{\hat{\theta}}_{i,j} & = & (\hat{\theta}_{i,j}^2 - \theta_M^2)\varpi_{i,j}(t,\tilde{\xi}), \quad 1 \leq i \leq s\,, 1 \leq j \leq p_i \\ \dot{\hat{\psi}}_i & = & \left(\hat{\psi}_i - \underline{\psi}\right)\left(\hat{\psi}_i - \overline{\psi}\right)\Upsilon_i(t,\tilde{\xi},\hat{\theta},\hat{\psi}), \quad 1 \leq i \leq s \end{array} \right. \tag{7}$$

Here $\hat{\theta}_i = (\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,p_i})$ for $i = 1, 2, \ldots, s$, $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R$,

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$$u_i(t, \tilde{\xi}, \hat{\theta}, \hat{\psi}) = \frac{v_{f,i}(t,\tilde{\xi}) - g_i(\xi) - k_i(\xi)\hat{\theta}_i + \dot{z}_{R,i}(t)}{\hat{\psi}_i} \tag{9}$$

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Barrier terms ensure $\underline{\psi} < \hat{\psi}_i(t) < \overline{\psi}$ and $|\hat{\theta}_{i,j}(t)| < \theta_M$ for all $t \geq 0$

Augmented Error Dynamics

Tracking error: $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R)$
Estimation errors: $\widetilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\widetilde{\psi}_i = \psi_i - \hat{\psi}_i$.

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$$\begin{array}{rcl} \mathcal{S} & = & \mathbb{R}^{r+s} \times \left(\prod_{i=1}^{s}\left\{\prod_{j=1}^{p_i}(\theta_{i,j} - \theta_M, \theta_{i,j} + \theta_M)\right\}\right) \\ & & \times\left(\prod_{i=1}^{s}\left(\psi_i - \overline{\psi}, \psi_i - \underline{\psi}\right)\right). \end{array}$$

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We build a strict LF for the augmented error dynamics for $\mathcal{E} = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ on its state space $\mathcal{S}$.

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We start with this nonstrict barrier type LF on $\mathcal{S}$:

$$
\begin{aligned}
V_1(t, \mathcal{E}) \; = \; & V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\widetilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \mathrm{d}m \\
& + \sum_{i=1}^{s} \int_0^{\widetilde{\psi}_i} \frac{m}{(\psi_i - m - \underline{\psi})(\overline{\psi} - \psi_i + m)} \mathrm{d}m \, .
\end{aligned}
$$

There is a positive definite function W such that $\dot{V}_1 \leq -W(\tilde{\xi})$ along all solutions $\mathcal{E} : [0, \infty) \to \mathcal{S}$ of (AED).

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We start with this nonstrict barrier type LF on $\mathcal{S}$:

$$
\begin{aligned}
V_1(t, \mathcal{E}) \;=\; & V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\widetilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \mathrm{d}m \\
& + \sum_{i=1}^{s} \int_0^{\widetilde{\psi}_i} \frac{m}{(\psi_i - m - \underline{\psi})(\overline{\psi} - \psi_i + m)} \mathrm{d}m \, .
\end{aligned}
$$

There is a positive definite function W such that $\dot{V}_1 \leq -W(\tilde{\xi})$ along all solutions $\mathcal{E} : [0, \infty) \to \mathcal{S}$ of (AED).

This allows $\dot{V}_1 = 0$ at some nonzero $\mathcal{E}$'s, so V_1 is nonstrict.

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We transform V_1 into the desired strict LF $V^\sharp$ for (AED).

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$V^\sharp$ enables proving ISS and rate of convergence analysis.

Our Transformation (M-M-dQ)

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Theorem: We can construct a function $\mathcal{L} \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\mathcal{E}) \;=\; \mathcal{L}\big(V_1(t,\mathcal{E})\big) + \sum_{i=1}^{s} \overline{\Omega}_i(t,\mathcal{E}) \;, \tag{10}$$

where

$$\overline{\Omega}_i(t,\mathcal{E}) = -\tilde{z}_i\lambda_i(t)\alpha_i(\mathcal{E}) + \tfrac{1}{T\overline{\psi}}\alpha_i^\top(\mathcal{E})\Omega_i(t)\alpha_i(\mathcal{E}) \;,$$

$$\alpha_i(\mathcal{E}) = \begin{bmatrix} \tilde{\theta}_i\psi_i - \theta_i\tilde{\psi}_i \\ \tilde{\psi}_i \end{bmatrix},\; \Omega_i(t) = \int_{t-T}^{t}\int_{m}^{t}\lambda_i^\top(s)\lambda_i(s)\mathrm{d}s\,\mathrm{d}m, \tag{11}$$

and $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$

is a strict LF for (AED) on its state space $\mathcal{S}$, so (AED) is UGAS.

One Application: Marine Robots

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We combined mathematical analysis with 2 weeks of field work with robotics students at a polluted lagoon at Grand Isle, LA.

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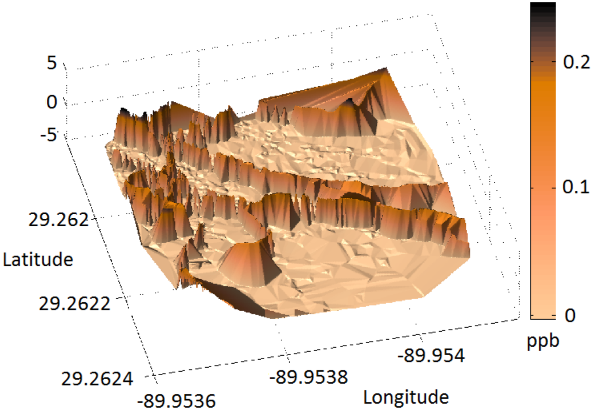

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Malisoff, M., and F. Zhang, "Robustness of adaptive control under time delays for three-dimensional curve tracking," *SIAM Journal on Control and Optimization*, 53(4):2203-2236, 2015.

Our Other Adaptive Control Applications

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Brushless DC motors turning a mechanical load with uncertain motor electric parameters including integral ISS analysis.

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Joint work with J. Muse from AFRL on model reference adaptive control to reduce oscillations, applied to hovering helicopters.

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Thanks for your interest!