

Adaptive Tracking and Parameter Identification

Michael Malisoff

Basic Problem Formulation

Consider a system of differential equations

$$\dot{\xi} = f(\xi, P, u) \quad (1)$$

with a vector P of **unknown constant** parameters and functions ξ_R and u_R such that $\dot{\xi}_R(t) = f(\xi_R(t), P, u_R(t))$ for all $t \geq 0$.

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Problem: Find $u(\xi, \hat{P})$ and a system of differential equations

$$\dot{\hat{P}} = g(\xi, \hat{P}) \quad (2)$$

such that with the control choice $u(\xi, \hat{P})$ in (1), all solutions $Y = (\tilde{\xi}, \tilde{P}) = (\xi - \xi_R, P - \hat{P})$ converge to 0 as $t \rightarrow +\infty$.

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This requires finding a control function u and a vector field g .

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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...

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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...

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Flight control, mechanical systems, robotics,...

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ZP. Jiang, E. Justh, P. Krishnaprasad, V. Lumelsky, A. Stepanov

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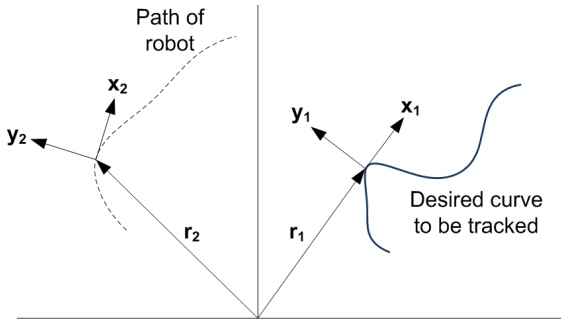
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Strict decay: there is a continuous positive definite α such that $\frac{d}{dt} V(Y(t)) \leq -\alpha(Y(t))$ along all solutions of system.

Gyroscopic Model (with Georgia Tech)

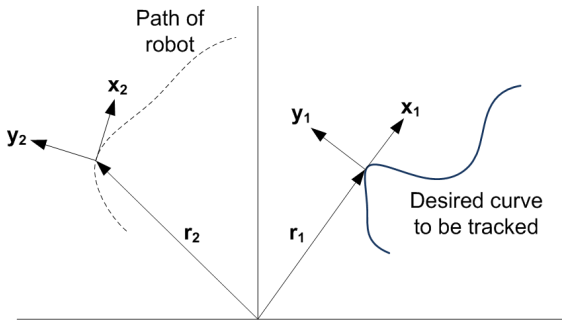
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Simpler 2D case: Boundary following with gyroscopic control.



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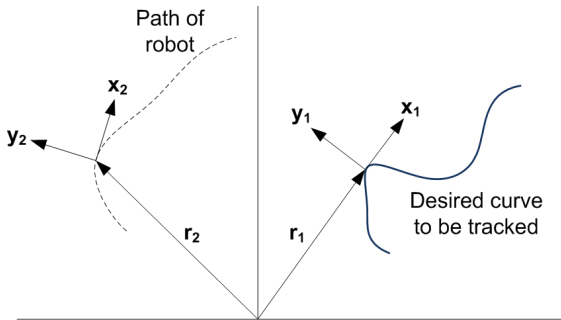
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Zhang-Justh-Krishnaprasad, IEEE-CDC'04.

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$$\rho = |\mathbf{r}_2 - \mathbf{r}_1|, \phi = \text{angle between } \mathbf{x}_1 \text{ and } \mathbf{x}_2, \cos(\phi) = \mathbf{x}_1 \cdot \mathbf{x}_2$$

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$$\begin{cases} \dot{\rho} = -\sin(\phi) \\ \dot{\phi} = \frac{\kappa \cos(\phi)}{1+\kappa\rho} - u_b, \end{cases} \quad (\rho, \phi) \in \mathcal{X} = (0, +\infty) \times (-\pi/2, \pi/2) \quad (3)$$

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Along all solutions of (CL) for all $t \geq 0$, we have $\frac{d}{dt} V(Y) \leq 0$.

Strict Lyapunov Function (Mazenc-M-Z, TAC)

Theorem 1: The closed loop system (CL) has the strict Lyapunov function

$$U(Y) =$$

$$-\textcolor{brown}{h}'(\rho) \sin(\phi) + \frac{1}{\mu} \int_0^{\textcolor{green}{V}(Y)} \gamma(m) dm + \Gamma(\textcolor{green}{V}(Y)) + \textcolor{green}{V}(Y),$$

$$\text{where } \gamma(q) = \frac{2(q+2\rho_0)^3}{\rho_0^4} + 1 + 0.5\mu^2 + \mu, \quad Y = (\rho - \rho_0, \phi),$$

$$\Gamma(q) = \frac{18}{\rho_0} q + 9 \left(\frac{2}{\rho_0} \right)^4 q^4, \text{ and } \textcolor{green}{V}(Y) = -\ln(\cos(\phi)) + \textcolor{brown}{h}(\rho)$$

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$$U(Y) \geq \textcolor{green}{V}(Y) \quad \text{(PD)}$$

$$\frac{d}{dt} U(Y) \leq -0.5[\textcolor{brown}{h}'(\rho) \cos(\phi)]^2 - \sin^2(\phi) \quad \text{(SD)}$$

Unknown Control Gains (M-Zhang)

$$\left\{ \begin{array}{l} \dot{\rho} = -\sin(\phi) \\ \dot{\phi} = \frac{\kappa \cos(\phi)}{1+\kappa\rho} + Ku, \quad K \in (c_{\min}, c_{\max}) \subseteq (0, \infty) \\ \dot{\hat{K}} = (\hat{K} - c_{\min})(c_{\max} - \hat{K}) \frac{\partial U}{\partial \phi} u, \quad \hat{K} \in (c_{\min}, c_{\max}) \end{array} \right. \quad (7)$$

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$$\begin{cases} \dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 &= \frac{\kappa \cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K}{\tilde{K}+K} u_b \\ \dot{\tilde{K}} &= -(\tilde{K} + K - c_{\min})(c_{\max} - \tilde{K} - K) \frac{\partial U}{\partial \phi} \frac{u_b}{\tilde{K}+K} \end{cases} \quad (8)$$

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i.e., the dynamics for $Y = (\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K)$. $\xi_R = (\rho_0, 0)$. Strictness allowed a robustness analysis to satisfy performance and safety bounds under other uncertainties.

Summer 2011 Field Work at Grand Isle, LA



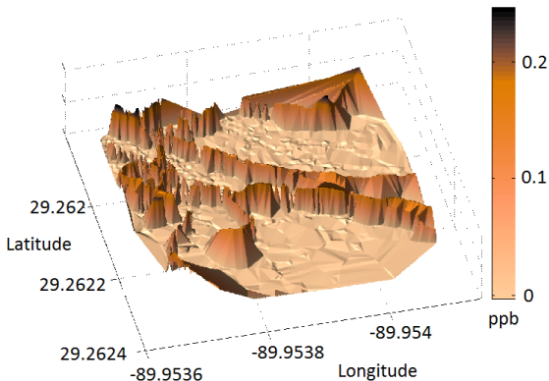
20 days of field work off Grand Isle. Search for oil spill remnants.
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Malisoff, M., and F. Zhang, "[Robustness of adaptive control under time delays for three-dimensional curve tracking](#)," *SIAM Journal on Control and Optimization*, 53(4):2203-2236, 2015.

Our Other Adaptive Control Applications

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Brushless DC motors turning a mechanical load with uncertain motor electric parameters including integral ISS analysis.

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Joint work with J. Muse from AFRL on model reference adaptive control to reduce oscillations, applied to hovering helicopters.

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Thanks for your interest!