Sequential Predictors for Input Delay Compensation in Control Systems

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Z. Artstein, I. Karafyllis, M. Krstic, S. Niculescu, P. Pepe, ...

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Assumption 1: The functions A and B are bounded and continuous, and there is a known bounded continuous function $K : [0, \infty) \to \mathbb{R}^{m \times n}$ such that $\dot{x}(t) = [A(t) + B(t)K(t)]x(t)$ is uniformly globally exponentially stable to 0.

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Assumption 2: The function $h : \mathbb{R} \to [0, \infty)$ is C^1 and bounded from above by a constant $c_h > 0$. Also, its derivative $\dot{h}$ is bounded from below, and $\dot{h}$ is bounded from above by a constant $l_h \in (0, 1)$, and $\dot{h}$ has a global Lipschitz constant $n_h > 0$.

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The control u will be specified by our theorem.

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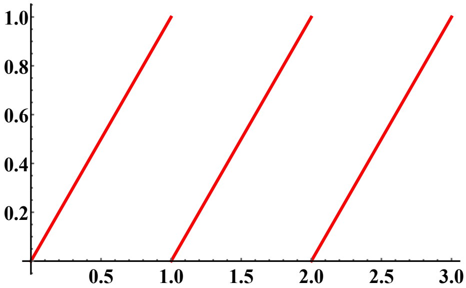

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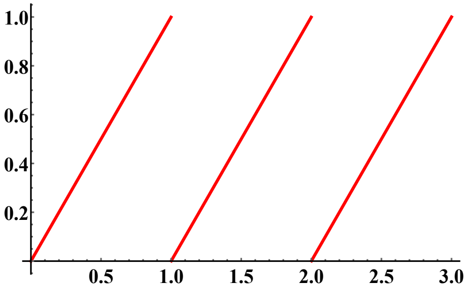

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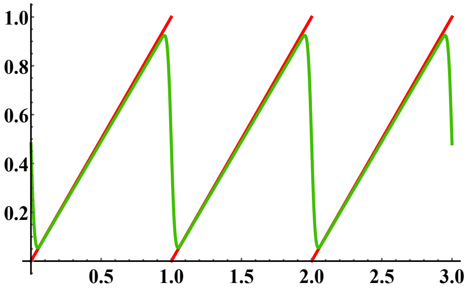

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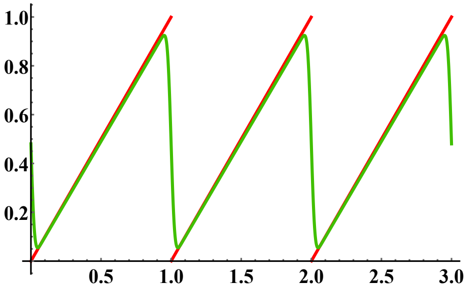

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Assumption 2 holds with $c_h = 0.924$, $l_h = 0.98$, and $n_h = 592.72$.

Preliminaries for Theorem

We use an pn-dimensional dynamic extension to build our delay compensating control for any number of predictors

$$p > \max\left\{2, 4\left(\frac{b_1}{\sqrt{2}} + b_2\right)\frac{c_h}{1-l_h}\right\}, \tag{LB}$$

where

$$b_1 = \left[1 + \left(1 + \frac{u_c}{p}\right)^p |A|_\infty\right]\left(1 + \frac{u_c}{p}\right)^p |A|_\infty,$$

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Theorem (*Automatica*, M and M, 2017)

Let Assumptions 1-2 hold and p satisfy (LB). Then if we use the control $u(t) = K(\Omega_p^{-1}(t))z_p(t)$ in (LTV), where z_p is the last n components of the system

$$\begin{aligned}
\dot{z}_1(t) &= R_1(t)A(\theta_1(t))z_1(t) + R_1(t)B(\theta_1(t))u(\Omega_{p-1}(t)) \\
&\quad + L_1(t)[z_1(\theta_1^{-1}(t)) - x(t)] \\
\dot{z}_i(t) &= R_i(t)A(G_i(t))z_i(t) + R_i(t)B(G_i(t))u(\Omega_{p-i}(t)) \\
&\quad + L_i(t)[z_i(\theta_i^{-1}(t)) - z_{i-1}(t)],\ i \in \{2, \dots, p\}
\end{aligned} \tag{1}$$

where $L_i(t) = -I_n - R_i(t)A(G_i(t))$ and $G_i = \Omega_p^{-1} \circ \Omega_{p-i}$, then the dynamics for $(x, \mathcal{E})$ are globally exponentially stable to 0, where $\mathcal{E}(t) = (z_1(t) - x(\theta_1(t)), z_2(t) - z_1(\theta_2(t)), \dots, z_p(t) - z_{p-1}(\theta_p(t)))$.

Pendulum Example

$$\begin{cases} \dot{r}_1(t) &= r_2(t) \\ \dot{r}_2(t) &= -\frac{g}{l}\sin(r_1(t)) + \frac{1}{\mathcal{M}l^2}v(t - h(t)) \end{cases} \tag{2}$$

Change of feedback and linearizing the tracking dynamics for tracking $(\omega t, \omega)$ for any $\omega > 0$ gives

$$\begin{cases} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -\frac{g}{l}\cos(\omega t)x_1(t) + u(t - h(t)) \end{cases} \tag{3}$$

Theorem applies with $h(t) = 1 + \alpha\sin(t)$ with $\alpha \in (0, 1)$.

E.g., if $l > g$ and $\omega > 0$ and $\alpha = 1/7$, can pick $p = 47$.

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Thank you for your attention!