Curve Tracking for Marine Robots: A Case Study in Feedback Control

Michael Malisoff, Louisiana State University

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$$Y'(t) = \mathcal{G}(t, Y(t), Y(t-\tau), \Gamma, \delta(t)), \quad Y(t) \in \mathcal{Y}, \qquad (2)$$

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Typically we construct u such that all trajectories of (2) for all possible choices of δ satisfy some control objective.

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Ex: When $\tau = 0$, Σ_{pert} is ISS iff it has an ISS Lf (Sontag-Wang).

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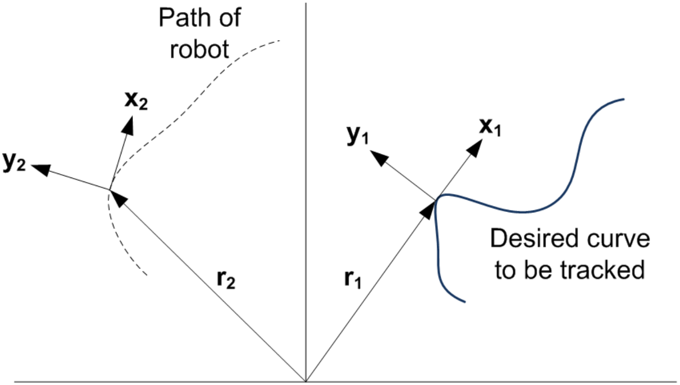

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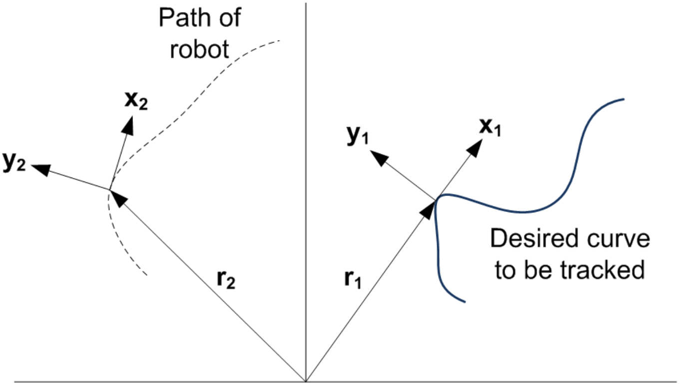

$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

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Feedback linearization with $z = \sin(\phi)$ cannot be applied.

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Strategy: Use the Lyapunov function candidate

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This gives global asymptotic stability, using LaSalle Invariance.

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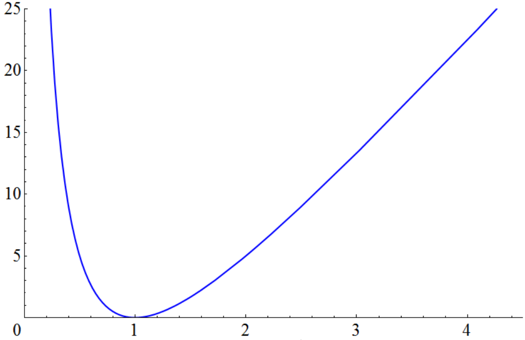

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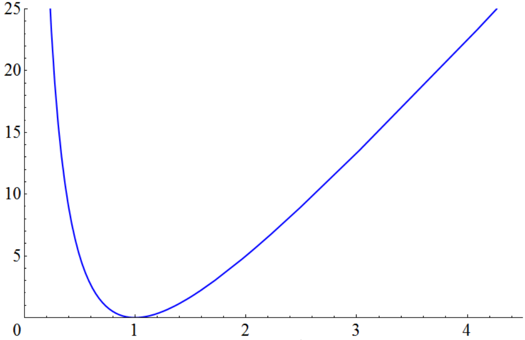

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See my Automatica and TAC papers with Fumin Zhang.

Our Adaptive Robust Curve Tracking Controller

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$$V^\sharp(\rho,\phi) = -h'(\rho)\sin(\phi) + \int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m \quad (8)$$

$$\gamma(q) = \frac{1}{\mu}\left(\frac{2}{\alpha^2\rho_0^4}(q+2\alpha\rho_0)^3 + 1\right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4\alpha^2}q^3 \quad (9)$$

$$V(\rho,\phi) = -\ln\left(\cos(\phi)\right) + h(\rho) \quad (10)$$

Robustly Forwardly Invariant Hexagonal Regions

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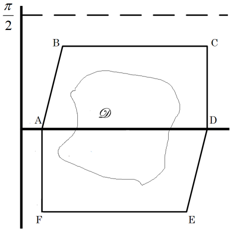

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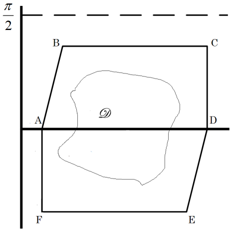

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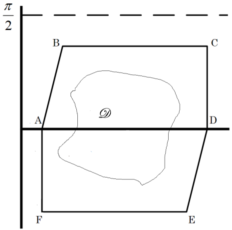

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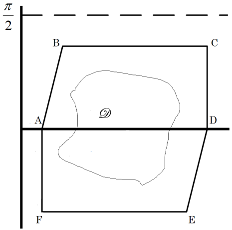

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For each index i, we take δ_{*i} to be the largest allowable disturbance bound to maintain forward invariance of H_i.

Then we prove ISS of the tracking and parameter identification dynamics for each set H_i and the disturbance set $\mathcal{D} = [-\delta_{*i}, \delta_{*i}]$.

Field Work at Grand Isle, LA

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20 days of field work off Grand Isle. Search for oil spill remnants. Georgia Tech Savannah Robotics Team (led by Fumin Zhang).

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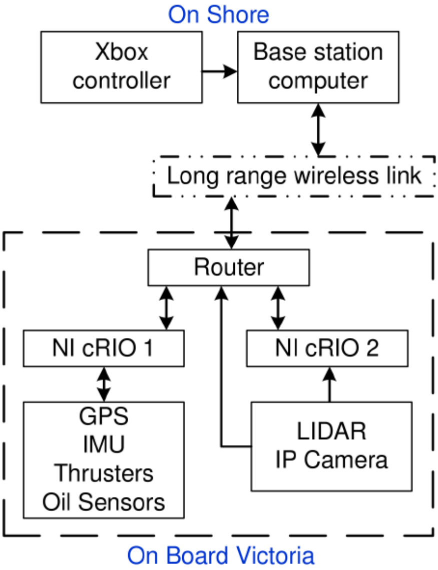
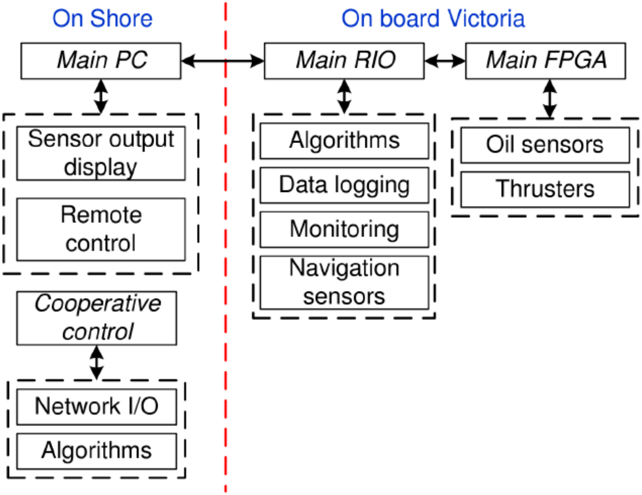
(Loading Video...)

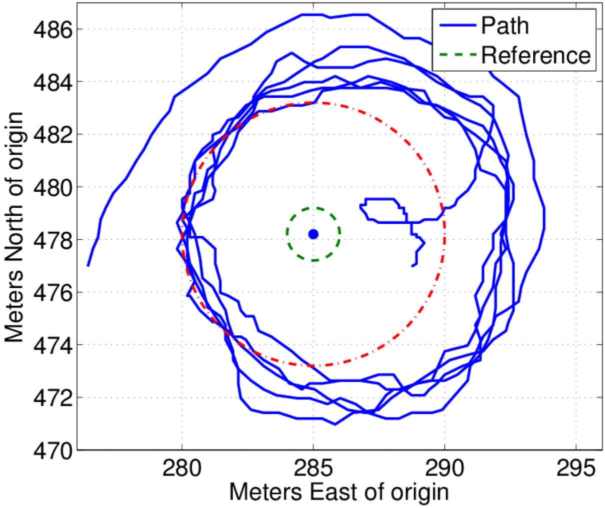
Schematic of ASV Victoria's Electrical Systems



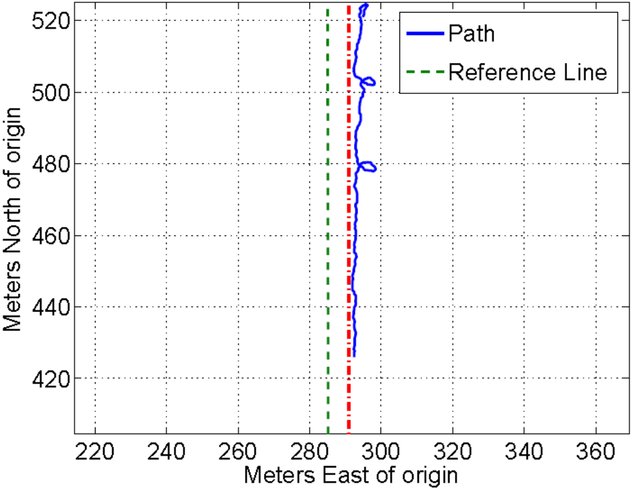
Schematic of ASV Victoria's Software Architecture



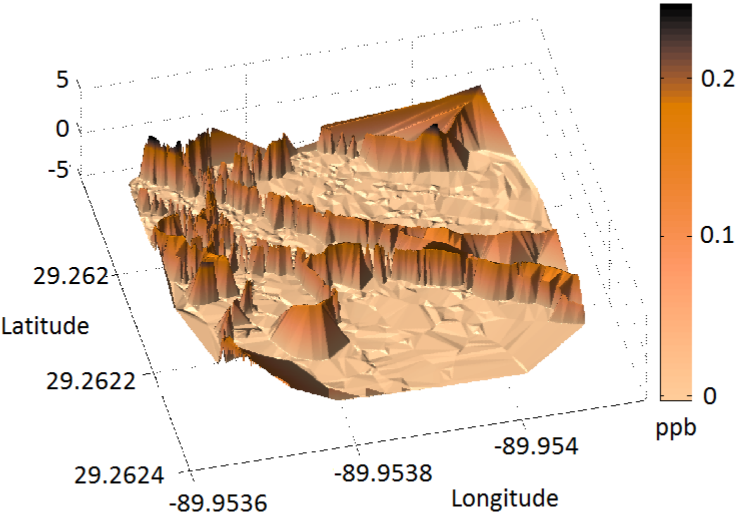
Circle Tracking by ASV Victoria



Line Tracking by ASV Victoria



Crude Oil Concentration Maps



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In our future work, we will study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.

References for 2D Case with Hyperlinks

Malisoff, M., F. Mazenc, and F. Zhang, “Stability and robustness analysis for curve tracking control using input-to-state stability," *IEEE Transactions on Automatic Control*, Volume 57, Issue 5, May 2012, pp. 1320-1326.

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References for 3D Case with Hyperlinks

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