Curve Tracking for Marine Robots: A Case Study in Feedback Control

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Joint with Georgia Tech Savannah Robotics Team

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What Do We Mean By Control Systems?

These are triply parameterized families of ODEs of the form

$$ Y'(t) = F(t, Y(t), u(t, Y(t-\tau)), \Gamma, \delta(t)) $$

where

$$ Y \subseteq \mathbb{R}^n $$

We have freedom to choose the control function $u$. The functions $\delta: [0, \infty) \to \mathbb{D}$ represent uncertainty. $\mathbb{D} \subseteq \mathbb{R}^m$. The vector $\Gamma$ is constant but unknown. $\tau$ is a constant delay. Specify $u$ to get a doubly parameterized closed loop family

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Ex: When \( \tau = 0, \Sigma_{\text{pert}} \) is ISS iff it has an ISS Lf (Sontag-Wang).
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Two-Dimensional Curve Tracking Model

\[ \dot{\rho} = -\sin \varphi, \quad \dot{\varphi} = \kappa \cos \varphi + \kappa \rho - u^2, \]

\[(\rho, \varphi) \in X.\]

\[\rho = \text{relative distance}, \quad \varphi = \text{bearing}.\]

\[X = (0, +\infty) \times (-\pi/2, \pi/2).\]

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Lumelsky-Stepanov.
Micaelli-Samson.
Morin-Samson.
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ISS:
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Review of Zhang-Justh-Krishnaprasad CDC’04

They realized Control Objective (A) using controllers of the form

\[ u_2 = \kappa \cos(\phi) + \kappa \rho - h'(\rho) \cos(\phi) + \mu \sin(\phi). \] (3)

Assumption 1:

\[ h : (0, +\infty) \to [0, \infty) \] is \( C^1 \), \( h' \) has only finitely many zeros, \( \lim_{\rho \to 0^+} h(\rho) = \lim_{\rho \to \infty} h(\rho) = \infty \), and \( h \in PD(\rho_0) \).

Strategy:

\[ V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho). \] (4)

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To realize our goals, we added assumptions on $h$ which hold for

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See my Automatica and TAC papers with Fumin Zhang.
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full state space \((\rho, \phi) \in (0, \infty) \times (-\pi/2, \pi/2)\) \((\Sigma_c)\)
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Control: \( u(\rho, \phi, \hat{\Gamma}) = -\frac{1}{\hat{\Gamma}} \left( \frac{\kappa \cos(\phi)}{1+\kappa \rho} - h'(\rho) \cos(\phi) + \mu \sin(\phi) \right) \) \quad (6)

Estimator: \( \dot{\hat{\Gamma}} = (\hat{\Gamma} - c_{\text{min}})(c_{\text{max}} - \hat{\Gamma}) \frac{\partial V^\#(\rho, \phi)}{\partial \phi} u(\rho, \phi, \hat{\Gamma}) \) \quad (7)

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\[
V^\#(\rho, \phi) = -h'(\rho) \sin(\phi) + \int_0^{V(\rho, \phi)} \gamma(m) dm  \quad (8)
\]

\[
\gamma(q) = \frac{1}{\mu} \left(\frac{2}{\alpha^2 \rho_0^4}(q + 2\alpha \rho_0)^3 + 1\right) + \frac{\mu}{2} + 2 + \frac{18\alpha}{\rho_0} + \frac{576}{\rho_0^4 \alpha^2} q^3  \quad (9)
\]

\[
V(\rho, \phi) = -\ln(\cos(\phi)) + h(\rho)  \quad (10)
\]
Robustly Forwardly Invariant Hexagonal Regions

We must restrict the suprema of the perturbations $\delta(t)$ to keep $(\rho, \phi)$ from exiting $X = (0, \infty) \times (-\pi/2, \pi/2)$.

View the state space $(0, \infty) \times (-\pi/2, \pi/2)$ as a union of compact hexagonally shaped regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$. For each $i$, all trajectories of $(\Sigma c)$ starting in $H_i$ for all $\delta : [0, \infty) \to [-\delta^*_i, \delta^*_i]$ stay in $H_i$. The tilted legs have slope $c_{\min} \mu / c_{\max}$.

For each index $i$, we take $\delta^*_i$ to be the largest allowable disturbance bound to maintain forward invariance of $H_i$.

Then we prove ISS of the tracking and parameter identification dynamics for each set $H_i$ and the disturbance set $D = [-\delta^*_i, \delta^*_i]$. 
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[Diagram with hexagonal regions $H_1 \subseteq H_2 \subseteq \ldots \subseteq H_i \subseteq \ldots$.]

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Field Work at Grand Isle, LA

20 days of field work off Grand Isle. Search for oil spill remnants.

Georgia Tech Savannah Robotics Team (led by Fumin Zhang).
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Schematic of ASV Victoria’s Electrical Systems

On Shore

- Xbox controller
- Base station computer

Long range wireless link

Router

On Board Victoria

- NI cRIO 1
  - GPS
  - IMU
  - Thrusters
  - Oil Sensors

- NI cRIO 2
  - LIDAR
  - IP Camera
Schematic of ASV Victoria’s Software Architecture
Circle Tracking by ASV Victoria
Line Tracking by ASV Victoria
Crude Oil Concentration Maps
Conclusions

Adaptive nonlinear controllers are useful for many engineering control systems with delays and uncertainties. Curve tracking controllers for autonomous marine vehicles are important for monitoring water quality, especially after oil spills. Our curve trackers are adaptive and robust to the perturbations and time delays that commonly arise in field work. We can prove these properties using input-to-state stability, parameter estimators, and Lyapunov-Krasovskii functionals. We used our controls on student built marine robots to map residual crude oil from the Deepwater Horizon spill. In our future work, we will study adaptive robust control for heterogeneous fleets of autonomous marine vehicles.
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References for 2D Case with Hyperlinks


