Adaptive Tracking and Parameter Identification

Michael Malisoff

Basic Problem Formulation

Consider a system of differential equations

$$\dot{\xi} \;=\; f(\xi, P, u) \tag{1}$$

with a vector P of unknown constant parameters and functions ξ_R and u_R such that $\dot{\xi}_R(t) = f(\xi_R(t), P, u_R(t))$ for all $t \geq 0$.

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Problem: Find $u(\xi, \hat{P})$ and a system of differential equations

$$\dot{\hat{P}} = g(\xi, \hat{P}) \tag{2}$$

such that with the control choice $u(\xi, \hat{P})$ in (1), all solutions $Y = (\tilde{\xi}, \tilde{P}) = (\xi - \xi_R, P - \hat{P})$ converge to 0 as $t \to +\infty$.

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Lavretsky-Wise, Narendra-Annaswamy, Sastry-Bodson,...

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Basar, Cortes, Dixon, Duncan, Krstic, Morse, Ortega, Yucelen,...

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Flight control, mechanical systems, robotics,...

Adaptive Robotic Curve Tracking

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ZP. Jiang, E. Justh, P. Krishnaprasad, V. Lumelsky, A. Stepanov

Gyroscopic Model (with Georgia Tech)

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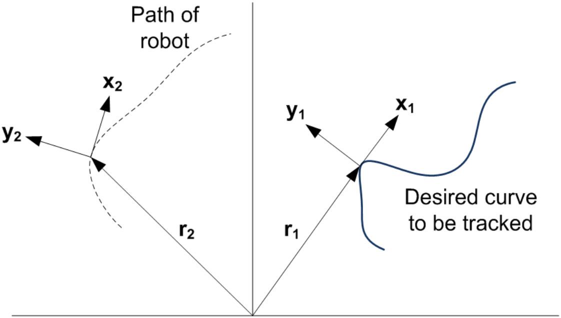
Simpler 2D case: Boundary following with gyroscopic control.



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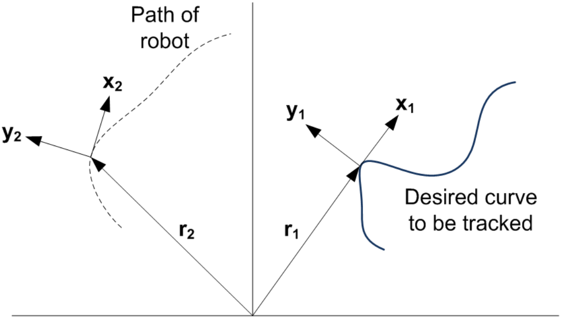

Zhang-Justh-Krishnaprasad, IEEE-CDC'04.

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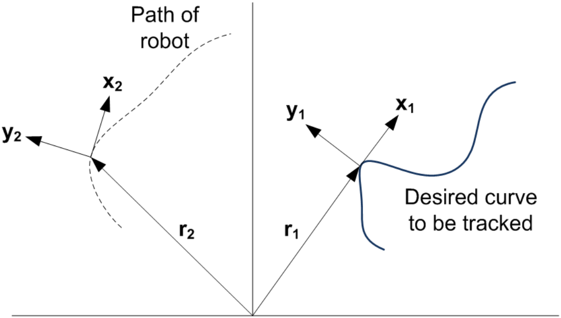

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$\rho = |\mathbf{r_2} - \mathbf{r_1}|$, $\phi =$ angle between $\mathbf{x_1}$ and $\mathbf{x_2}$, $\cos(\phi) = \mathbf{x_1} \cdot \mathbf{x_2}$

Curve Tracking Dynamics for 2D

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Along all solutions of (CL) for all $t \geq 0$, we have $\frac{d}{dt}V(\rho,\phi) \leq 0$.

Strict Lyapunov Function (Mazenc-M-Z, TAC)

Theorem 1: The closed loop system (CL) has the strict Lyapunov function

$$U(\rho,\phi) =$$
$$-h'(\rho)\sin(\phi) + \frac{1}{\mu}\int_0^{V(\rho,\phi)} \gamma(m)\mathrm{d}m + \Gamma(V(\rho,\phi)) + V(\rho,\phi),$$

where $\gamma(q) = \frac{2(q+2\rho_0)^3}{\rho_0^4} + 1 + 0.5\mu^2 + \mu$,

$\Gamma(q) = \frac{18}{\rho_0}q + 9\left(\frac{2}{\rho_0}\right)^4 q^4$, and $V(\rho,\phi) = -\ln\big(\cos(\phi)\big) + h(\rho)$

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on its state space $\mathcal{X} = (0,+\infty) \times (-\pi/2, \pi/2)$. $\square$

$$U(\rho(t),\phi(t)) \geq V(\rho(t),\phi(t)) \qquad \text{(PD)}$$

$$\frac{d}{dt} U(\rho(t),\phi(t)) \leq -0.5[h'(\rho(t))\cos(\phi(t))]^2 - \sin^2(\phi(t)) \qquad \text{(SD)}$$

Unknown Control Gains (M-Zhang)

$$\begin{cases} \dot{\rho} &= -\sin(\phi) \\ \dot{\phi} &= \frac{\kappa\cos(\phi)}{1+\kappa\rho} + Ku, \quad K \in (c_{\min}, c_{\max}) \subseteq (0, \infty) \\ \dot{\hat{K}} &= (\hat{K} - c_{\min})(c_{\max} - \hat{K})\frac{\partial U}{\partial \phi}u, \quad \hat{K}(0) \in (c_{\min}, c_{\max}) \end{cases} \tag{7}$$

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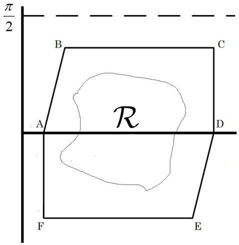
$u(\rho, \phi, \hat{K}) = -u_b(\rho, \phi)/\hat{K}$. Proved input-to-state stability to 0 for

$$\begin{cases} \dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 &= \frac{\kappa\cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K}{\tilde{K}+K}u_b + \delta(t) \\ \dot{\tilde{K}} &= -(\tilde{K} + K - c_{\min})(c_{\max} - \tilde{K} - K)\frac{\partial U}{\partial \phi}\frac{u_b}{\tilde{K}+K} \end{cases} \tag{8}$$

for the augmented error $(\tilde{q}_1, \tilde{q}_2, \tilde{K}) = (\rho - \rho_0, \phi, \hat{K} - K)$ on each set $\mathcal{S}_i \times (c_{\min} - K, c_{\max} - K)$ where $\mathcal{S}_i$ is a nested sequence of compact sets that fill our state space $\mathcal{X}$.

Nested Hexagons that Fill $\mathcal{X}$



AB and DE have slope μ

Others flat or vertical

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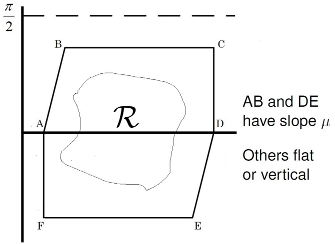

$A = (\rho_*, 0)^\top$, $B = (2\rho_*, \mu\rho_*)^\top$, $C = (\rho_* + \bar{K}\rho_0, \mu\rho_*)^\top$,
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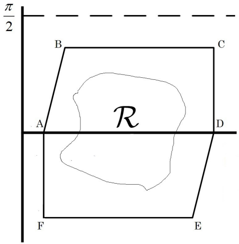

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For each hexagon $\mathcal{H}$, we compute the sup of all $\bar{\Delta}$'s such that

$$\dot{\rho} = -\sin(\phi), \quad \dot{\phi} = h'(\rho)\cos(\phi) - \mu\sin(\phi) + \delta(t) \qquad (\mathrm{CL}_\delta)$$

has $\mathcal{H}$ as a forward invariant set for all $\delta : [0, \infty) \to [-\bar{\Delta}, \bar{\Delta}]$.

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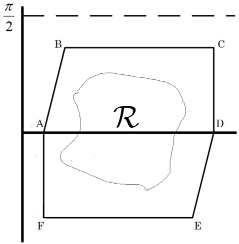

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I.e., all solutions of (CL_δ) starting in $\mathcal{H}$ stay in $\mathcal{H}$ if $\sup_t |\delta(t)| \leq \bar{\Delta}$

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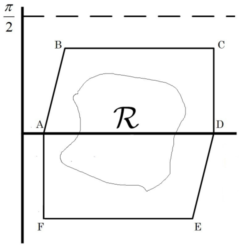

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Robust forward invariant sets with maximum perturbation sets.

Barrier Lyapunov Function and Matrosov Ideas

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$$U^\sharp(\tilde{q},\tilde{K}) = U\big(\tilde{q}+\mathcal{E}\big) + \int_0^{\tilde{K}} \frac{\ell}{(\ell+K-c_{\min})(c_{\max}-\ell-K)} \mathrm{d}\ell\ . \tag{10}$$

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$$\text{Strictification:}\ \ V^\sharp(\tilde{q},\tilde{K}) \ = \ c_* U^\sharp(\tilde{q},\tilde{K}) + \tilde{K}^2 - \tilde{q}_2\tilde{K} \tag{12}$$

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Strict Decay: $\dot{V}^\sharp \leq -\alpha_0(V^\sharp)$, with α_0 positive definite (13)

Summer 2011 Field Work at Grand Isle, LA

20 days of field work off Grand Isle. Search for oil spill remnants.
Georgia Tech Savannah Robotics (co-led by Fumin Zhang)

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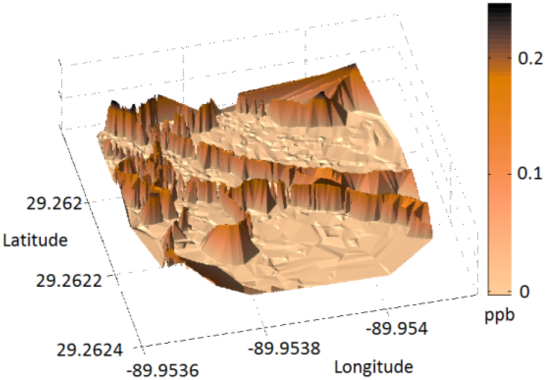

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Hyperlinked Related References

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Our Other Adaptive Control Applications

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Joint work with J. Muse from AFRL on model reference adaptive control to reduce oscillations, applied to hovering helicopters.

Conclusions

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Adaptive nonlinear controllers are useful for many engineering control systems for which parameters need to be identified.

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Thanks for your interest!

Backup Slides to Use if Time Allows or Questions Warrant

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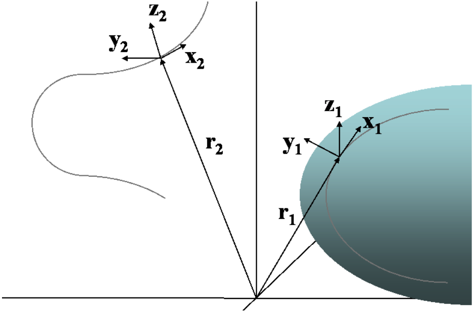
Ideas from:
Malisoff, M., and F. Zhang, "Robustness of adaptive control under time delays for three-dimensional curve tracking," *SIAM Journal on Control and Optimization*, 53(4):2203-2236, 2015.

3D Curve Tracking by Unit Speed Robot



$$
\begin{aligned}
\dot{\mathbf{r}}_1 &= \alpha\,\mathbf{x}_1 \\
\dot{\mathbf{x}}_1 &= \alpha\kappa_n\,\mathbf{y}_1 + \alpha\kappa_g\,\mathbf{z}_1 \\
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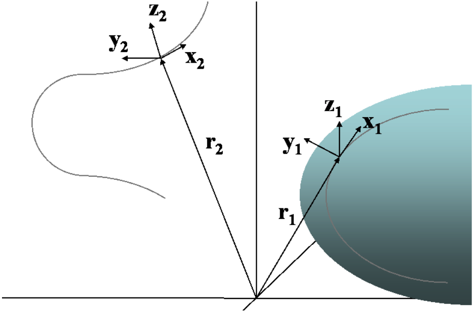

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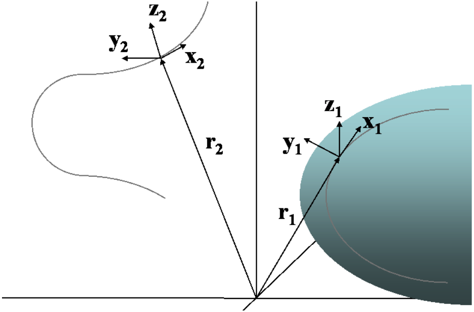

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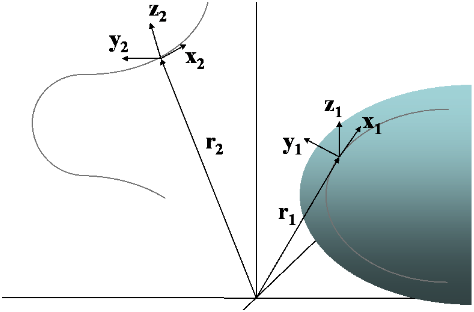

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Basic Control Goal: Find u and v such that $|\mathbf{r_1}(t) - \mathbf{r_2}(t)| \rightarrow \rho_c$ for a desired $\rho_c > 0$ and $\mathbf{x_1} \cdot \mathbf{x_2} \rightarrow 1$, while compensating for additive uncertainty and delays and identifying control gains.

Our New Variables and Control Design

$(\rho_1, \rho_2) = ((\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{y}_1, (\mathbf{r}_2 - \mathbf{r}_1) \cdot \mathbf{z}_1)$ has desired value (ρ_{c1}, ρ_{c2}).

$\rho_c = |(\rho_{c1}, \rho_{c2})|$. Shape vars: $\varphi = \mathbf{x}_1 \cdot \mathbf{x}_2$, $\beta = \mathbf{y}_1 \cdot \mathbf{x}_2$, $\gamma = \mathbf{z}_1 \cdot \mathbf{x}_2$

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$$\begin{aligned} u &= a_1(\mathbf{x}_1 \cdot \mathbf{y}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{y}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{y}_2), \\ v &= a_1(\mathbf{x}_1 \cdot \mathbf{z}_2) + a_2(\mathbf{y}_1 \cdot \mathbf{z}_2) + a_3(\mathbf{z}_1 \cdot \mathbf{z}_2), \\ a_1 &= \mu,\ a_2 = -h_1'(\rho_1) + \tfrac{\alpha\kappa_n}{\varphi},\ a_3 = -h_2'(\rho_2) + \tfrac{\alpha\kappa_g}{\varphi},\ \text{and} \qquad (14) \\ h_i(\rho_i) &= \begin{cases} \bar{c}\left(\rho_i + \rho_{ci}^2/\rho_i - 2\rho_{ci}\right), & \rho_i \in (0, \rho_{ci}) \\ \frac{\bar{c}}{\rho_{ci}}(\rho_i - \rho_{ci})^2, & \rho_i \geq \rho_{ci} \end{cases} \end{aligned}$$

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New state $Y = (\rho_1, \zeta, \rho_2, \theta)$ takes its values in $\mathcal{X}^\sharp$, where $(\varphi, \beta, \gamma) = \big(\cos(\zeta)\cos(\theta), -\sin(\zeta)\cos(\theta), \sin(\theta)\big)$ and where $\mathcal{X}^\sharp = (0,\infty) \times (-\pi/2, \pi/2) \times (0,\infty) \times (-\pi/2, \pi/2)$.

A Key Ingredient: Strict Lyapunov Function

$$
\begin{aligned}
\dot{\rho}_1 &= -\sin(\zeta)\cos(\theta) \\
\dot{\zeta} &= -\frac{1}{\cos^2(\theta)}\Big[\alpha\kappa_n \sin^2(\theta) - h_1'(\rho_1)\cos(\zeta)\cos(\theta) \\
&\quad + \alpha\kappa_g \sin(\theta)\sin(\zeta)\cos(\theta) + \mu\sin(\zeta)\cos(\theta)\Big] \\
\dot{\rho}_2 &= \sin(\theta) \\
\dot{\theta} &= \alpha\kappa_g \frac{\sin^2(\zeta)}{\cos(\zeta)} - h_2'(\rho_2)\cos(\theta) - \mu\cos(\zeta)\sin(\theta) \\
&\quad + \left(-h_1'(\rho_1) + \frac{\alpha\kappa_n}{\cos(\theta)\cos(\zeta)}\right)\sin(\zeta)\sin(\theta)
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Theorem (MZ, SICON'15): We can build a function $\mathcal{L}$ such that

$$U(Y) = -h_1'(\rho_1)\sin(\zeta)\cos(\theta) + h_2'(\rho_2)\sin(\theta) + \int_0^{V(Y)} \mathcal{L}(q)\mathrm{d}q$$

is a strict Lyapunov function for (15) on $\mathcal{X}^\sharp$ for the equilibrium $(\rho_{c1}, 0, \rho_{c2}, 0)$, where $V(Y) = -\ln(\cos(\theta)\cos(\zeta)) + h_1(\rho_1) + h_2(\rho_2)$.

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