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★**Constructions of strict Lyapunov functions.**

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The aim of this book is to present a systematic design approach for the construction of an explicit strict Lyapunov function provided that a non-strict Lyapunov function is known. In Chapter 1 some basic concepts from the theory of nonlinear control systems are presented, including stability and input-to-state stability (ISS) as well as an overview of the feedback stabilization problem. Chapter 2 contains characterizations of various concepts of asymptotic stability, ISS and integral ISS (iISS), by means of strict and non-strict Lyapunov functions. It also contains precise statements of Matrosov's theorem, the LaSalle invariance principle for asymptotic stability and the Jurdjevic-Quinn theorem for feedback stabilization.

In Chapter 3 several methods are presented for constructing strict Lyapunov functions for time-invariant systems that satisfy appropriate Matrosov conditions. The approach involves transformation of a given non-strict Lyapunov function into an explicit global control Lyapunov function. This gives a strict Lyapunov function construction for closed-loop Jurdjevic-Quinn systems with feedbacks of arbitrarily small magnitude. The corresponding results are illustrated by a two-link manipulator model, as well as an integral ISS result.

Chapter 5 contains two more methods for constructing strict Lyapunov functions, which apply to cases where asymptotic stability is already known from the LaSalle invariance principle. The first method imposes certain algebraic conditions on the higher-order Lie derivatives of the non-strict Lyapunov functions, along the trajectories of the dynamical system. The second method uses the continuous-time Matrosov theorem. The approach is illustrated by constructing a strict Lyapunov function for the error dynamics involving a Lotka-Volterra predator-prey system.

Chapter 6 contains some basic results on strictification of Lyapunov functions as well as of control Lyapunov functions for the case of time-varying systems. The results are applied to stabilization problems for rotating rigid bodies and underactuated ships. Chapter 7 begins with a review of classical backstepping for time-invariant systems. Several extensions are then provided that lead to time-varying strict Lyapunov functions and stabilizing feedbacks for time-varying systems. Among other things, nonlinear systems in feedback form are studied, and sufficient conditions for the global uniform stabilizability by bounded control laws are established.

In Chapter 8 a general strict Lyapunov function construction for time-varying systems is established, under a set of Matrosov-type hypotheses. The construction is applied to systems that satisfy time-varying versions of the Jurdjevic-Quinn and LaSalle conditions. The results are illustrated by considering a stabilization problem for a time-varying system with a sign constrained controller. In Chapter 9 a generalization of strictification is presented for the case of time-varying systems whose dynamics contain unknown parameters. The method involves constructing global strict Lyapunov functions for an augmented system that includes the tracking error and the parameter estimation error. The strictification approach makes it possible to quantify the effects of other types of uncertainty in the model as well, using the input-to-state stability framework.

In Chapter 10 a systematic method for rapidly time-varying systems is provided. The method involves transforming Lyapunov functions for the corresponding limiting dynamics into the desired strict Lyapunov functions for the original rapidly time-varying dynamics. Chapter 11 deals with the problem of explicitly constructing strict Lyapunov functions for slowly time-varying systems. The corresponding techniques are completely different from the ones in Chapter 10; instead of using limiting dynamics or averaging, a frozen dynamics approach is applied, whereby Lyapunov functions are used to build strict Lyapunov functions for the original slowly time-varying dynamics. The results are illustrated using friction and pendulum models.

In Chapter 12 some results on constructive nonlinear control for hybrid systems are presented that provide explicit ISS Lyapunov functions in terms of given non-strict Lyapunov functions for the continuous and discrete subsystems, as well as a hybrid version of Matrosov's theorem.

Reviewed by *John Tsinias*