Feedback Control under Input Delays

Michael Malisoff

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Delays and Sampling: Time-lagged state observations and/or observations at discrete instants instead of continuous ones.

Control Systems with Input Delays

System of ODEs with delays τ, controls u, and perturbations δ:

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t-\tau(t))), \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. $\delta : [0, \infty) \to \mathcal{D}$ is (nonstochastic) uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
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Closed loop system:

$$Y'(t) = \mathcal{G}(t, Y(t), Y(t - \tau(t)), \delta(t)), \quad Y(t) \in \mathcal{Y}, \qquad (2)$$

where $\mathcal{G}(t, Y(t), Y(t - \tau), d) = \mathcal{F}(t, Y(t), u(t, Y(t - \tau)), d)$.

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$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t} \gamma_2(|Y|_{[t_0 - \bar{\tau}, t_0]}) \right) \tag{UGAS}$$

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Without explicit flow maps, prove UGAS and ISS indirectly.

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O. Bernard, D. Dochain, J. Gouze, J. Monod, H. Smith

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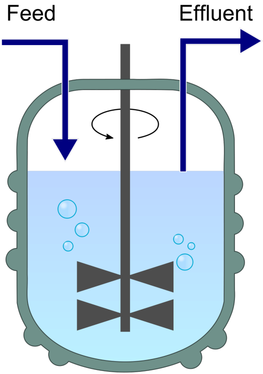

Constant volume. Substrate pumped in and substrate/biomass mixture pumped out at same rate

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Goal: Under suitable conditions and constants $s_* \in (0, s_{\rm in})$, find D to render the dynamics for $Y(t) = (s,x)(t) - (s_*, s_{\rm in} - s_*)$ ISS.

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$$\varpi_s = \inf_{s\in[0,s_{\text{in}}]} \mu_1'(s)\ ,\ \ \varpi_l = \sup_{s\in[0,s_{\text{in}}]} \mu_1'(s)\ ,\ \ \rho_l = \sup_{s\in[0,s_{\text{in}}]} \gamma'(s),$$

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Assume that $\frac{\mu_1(s_{\rm in})}{1+\gamma(s_{\rm in})} - \frac{\mu_1(s_*)}{1+\gamma(s_{\rm in}-\mu_1(s_*)s_{\rm in}\bar{\tau})} > 0$

and $\bar{\tau} < \max\left\{\frac{1}{2\sqrt{2\rho_m\varpi_l}s_{\rm in}}, \frac{1}{2\rho_l s_{\rm in}\mu_1(s_{\rm in})}\right\}$, with $s_* < s_{\rm in}$.

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Theorem: For all componentwise positive initial conditions, all solutions of the chemostat system (C) with $\delta(t) = 0$ and

$$D(s(t-\tau(t))) = \frac{\mu_1(s_*)}{1+\gamma(s(t-\tau(t)))} \qquad (3)$$

remain in $(0,\infty)^2$ and converge to $(s_*, s_{\rm in} - s_*)$. □

Extensions and Ideas of Proof

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ISS with respect to $\delta(t)$ without upper bounds on $|\delta|_\infty$...

$$\frac{(1+\underline{d})\mu_1(s_{\text{in}})}{1+\gamma(s_{\text{in}})} - \frac{\mu_1(s_*)}{1+\gamma(s_{\text{in}}-\mu_1(s_*)s_{\text{in}}\bar{\tau})} > 0 \quad ...(1+\delta(t))\mu(s(t))...$$

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$$\mathcal{U}_2(s_t) = \int_0^{s(t)-s_*} \frac{m}{s_{\mathrm{in}}-s_*-m}\mathrm{d}m + 2\rho_m\bar{\tau}\int_{t-\bar{\tau}}^{t}\int_{\ell}^{t}(\dot{s}(m))^2\mathrm{d}m\,\mathrm{d}\ell.$$

Combine with the fact that $z = s_{\mathrm{in}} - s - x \to 0$ exponentially.

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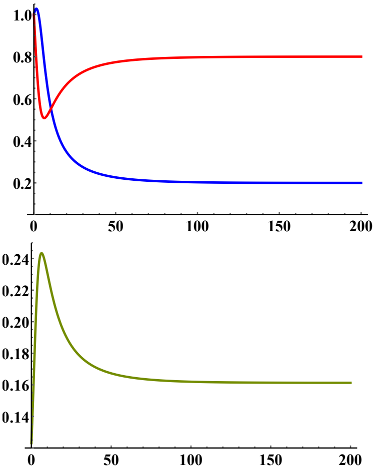
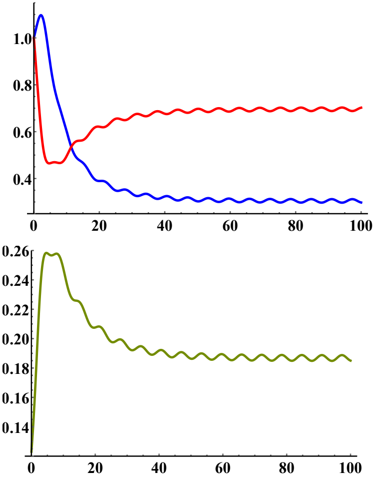
Mazenc, F., J. Harmand, and M. Malisoff, “Stabilization in a chemostat with sampled and delayed measurements and uncertain growth functions,” *Automatica*, 78:241-249, 2017.



$$s_{\text{in}} = 1,\ \mu(s) = \frac{0.5s}{1+0.25s+2s^2},\ t_j = 0.24j,\ \delta(t) = 0.$$

$s(t)$ in Red, $x(t)$ in Blue, $D(t)$ in Green.



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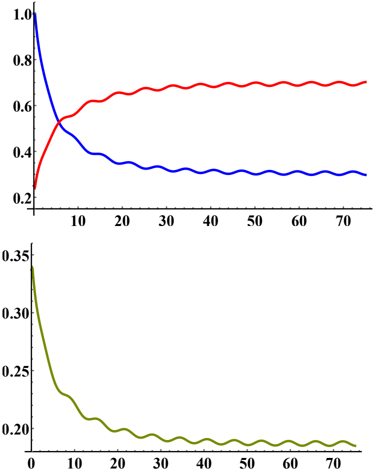

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Thank you for your attention!