Feedback Control under Input Delays

Michael Malisoff
Background on Research Area

Systems and Controls: Gives methods to influence the behavior of complicated dynamical systems to achieve objectives.

Open Loop Control: Time-dependent forcing functions chosen to optimize or achieve desired behavior for system's solutions.

Feedback Control: Automatically adjust the system to respond to information about the system's state and surroundings.

Delays and Sampling: Time-lagged state observations and/or observations at discrete instants instead of continuous ones.
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Control Systems with Input Delays

System of ODEs with delays $\tau$, controls $u$, and perturbations $\delta$:

\[ Y'(t) = F(t, Y(t), u(t, Y(t - \tau(t))), \delta(t)), \quad Y(t) \in \mathcal{Y}. \] (1)

$\mathcal{Y} \subseteq \mathbb{R}^n$. $\delta : [0, \infty) \rightarrow \mathcal{D}$ is (nonstochastic) uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$. Choose $u$ to achieve desired behavior for the solutions $Y(t)$. 
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Closed loop system:

$$Y'(t) = \mathcal{G}(t, Y(t), Y(t - \tau(t)), \delta(t)), \quad Y(t) \in \mathcal{Y}, \quad (2)$$

where $\mathcal{G}(t, Y(t), Y(t - \tau), \delta) = \mathcal{F}(t, Y(t), u(t, Y(t - \tau)), \delta)$. 
Input-to-State Stable (ISS)
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ISS (Sontag, '89) generalizes uniform global asymptotic stability.
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\[ Y'(t) = G(t, Y(t), Y(t - \tau(t))), \quad Y(t) \in \mathcal{Y} \]  

(Σ)

Without explicit flow maps, prove UGAS and ISS indirectly.
Input-to-State Stable (ISS)

ISS (Sontag, ’89) generalizes uniform global asymptotic stability.

\[ Y'(t) = G(t, Y(t), Y(t - \tau(t))), \quad Y(t) \in \mathcal{Y} \quad \text{(Σ)} \]

\[ |Y(t)| \leq \gamma_1 (e^{t_0 - t} \gamma_2 (|Y|_{[t_0 - \bar{\tau}, t_0]})) \quad \text{(UGAS)} \]

\( \gamma_i \)'s are 0 at 0, strictly increasing, continuous, and unbounded.

\[ \sup_{t \geq 0} \tau(t) \leq \bar{\tau}. \]
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\[ |Y(t)| \leq \gamma_1 (e^{t_0-t} \gamma_2(|Y|_{[t_0-\bar{\tau}, t_0]})) + \gamma_3(|\delta|_{[t_0,t]}) \quad (ISS) \]
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O. Bernard, D. Dochain, J. Gouze, J. Monod, H. Smith
Background on Chemostats

Constant volume. Substrate pumped in and substrate/biomass mixture pumped out at same rate
Uncertain Controlled Chemostat with Sampling

\[ \dot{s}(t) = D(s(t - \tau(t)))[s_{\text{in}} - s(t)] - (1 + \delta(t))\mu(s(t))x(t) \]

\[ \dot{x}(t) = [(1 + \delta(t))\mu(s(t)) - D(s(t - \tau(t)))]x(t) \]

\( s = \text{substrate concentration}, \quad x = \text{species concentration} \)

\( Y = (0, \infty)^2 \)

\( \tau(t) = t - t_j \text{ if } t \in [t_j, t_j + 1) \) and \( j \geq 0 \)

\( 0 < \epsilon \leq t_{i+1} - t_i \leq \bar{\tau} \).

\( \delta: [0, \infty) \rightarrow [\delta_0, \infty), \text{ with } \delta_0 \in (-1, 0) \).

\( \mu(s) = \mu_1(s) + \gamma(s) \), with a unique maximizer \( s_M \in (0, s_{\text{in}}] \).

Goal: Under suitable conditions and constants \( s^* \in (0, s_{\text{in}}) \), find \( D \) to render the dynamics for \( Y(t) = (s, x)(t) - (s^*, s_{\text{in}} - s^*) \) ISS.
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\begin{aligned}
\dot{s}(t) &= D(s(t - \tau(t))[s_{\text{in}} - s(t)] - (1 + \delta(t))\mu(s(t))x(t) \\
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\]  
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One of My Results for Unperturbed Case
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\[ \varpi_s = \inf_{s \in [0, s_{\text{in}}]} \mu_1'(s), \quad \varpi_l = \sup_{s \in [0, s_{\text{in}}]} \mu_1'(s), \quad \rho_l = \sup_{s \in [0, s_{\text{in}}]} \gamma'(s), \]

\[ \rho_m = \frac{\rho_l^2}{2\varpi_s} \max_{l \in [0, s_{\text{in}}]} \frac{\mu_1^2(l + 1.1 \mu_1(s_*) s_{\text{in}})}{1 + \gamma(l)}, \quad \text{where} \quad \mu(s) = \frac{\mu_1(s)}{1 + \gamma(s)} \]
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Assume that

\[ \frac{\mu_1(s_{in})}{1 + \gamma(s_{in})} - \frac{\mu_1(s_*)}{1 + \gamma(s_{in} - \mu_1(s_*) s_{in} \bar{\tau})} > 0 \]

and \( \bar{\tau} < \max \left\{ \frac{1}{2\sqrt{2 \rho_m \varpi_s s_{in}}}, \frac{1}{2 \rho_l s_{in} \mu_1(s_{in})} \right\} \), with \( s_* < s_{in} \).
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Assume that

\[ \frac{\mu_1(s_{in})}{1 + \gamma(s_{in})} \left( 1 + \frac{\mu_1(s_*)}{1 + \gamma(s_{in} - \mu_1(s_*) s_{in} \bar{\tau})} \right) > 0 \]

and \( \bar{\tau} < \max \left\{ \frac{1}{2 \sqrt{2 \rho_m \varpi_s s_{in}}}, \frac{1}{2 \rho_l s_{in} \mu_1(s_{in})} \right\} \), with \( s_* < s_{in} \).

**Theorem:** For all componentwise positive initial conditions, all solutions of the chemostat system (C) with \( \delta(t) = 0 \) and

\[ D(s(t - \tau(t))) = \frac{\mu_1(s_*)}{1 + \gamma(s(t - \tau(t)))} \]

remain in \((0, \infty)^2\) and converge to \((s_*, s_{in} - s_*)\). \( \square \)
Extensions and Ideas of Proof
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ISS with respect to $\delta(t)$ without upper bounds on $|\delta|_\infty$...

\[
\frac{(1+d)\mu_1(s_{in})}{1+\gamma(s_{in})} - \frac{\mu_1(s_*)}{1+\gamma(s_{in}-\mu_1(s_*)s_{in}\bar{\tau})} > 0 \quad ... (1 + \delta(t))\mu(s(t)) ...
\]
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\]

\[U_2(s_t) =\]
\[
\int_0^{s(t)-s_*} \frac{m}{s_{\text{in}}-s_*-m} \, dm + 2\rho m \bar{\tau} \int_{t-\bar{\tau}}^t \int_{\ell}^t (\dot{s}(m))^2 \, dm \, d\ell.
\]

Combine with the fact that $z = s_{\text{in}} - s - x \to 0$ exponentially.
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\[
U_2(s_t) = \\
\int_{s(t)-s_*}^{s(t)} \frac{m}{s_{in}-s_*-m} \, dm + 2\rho m\bar{\tau} \int_{t-\bar{\tau}}^{t} \int_{\ell}^{t} (s(m))^2 \, dm \, d\ell.
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Combine with the fact that $z = s_{in} - s - x \rightarrow 0$ exponentially.

\( s_{in} = 1, \quad \mu(s) = \frac{0.5s}{1+0.25s+2s^2}, \quad t_j = 0.24j, \quad \delta(t) = 0. \)

\( s(t) \) in Red, \( x(t) \) in Blue, \( D(t) \) in Green.
$s_{in} = 1, \quad \mu(s) = \frac{0.5s}{1 + 0.25s + 2s^2}, \quad t_j = 0.24j, \quad \delta(t) = 0.15(1 + \sin(t))$.

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Thank you for your attention!