Feedback Control under Input Delays

Michael Malisoff

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Delays and Sampling: Time-lagged state observations and/or observations at discrete instants instead of continuous ones.

Control Systems with Input Delays

System of ODEs with delays τ, controls u, and perturbations δ:

$$Y'(t) = \mathcal{F}\big(t, Y(t), u(t, Y(t-\tau(t))), \delta(t)\big), \quad Y(t) \in \mathcal{Y}. \qquad (1)$$

$\mathcal{Y} \subseteq \mathbb{R}^n$. $\delta : [0, \infty) \to \mathcal{D}$ is (nonstochastic) uncertainty. $\mathcal{D} \subseteq \mathbb{R}^m$.
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Closed loop system:

$$Y'(t) = \mathcal{G}(t, Y(t), Y(t - \tau(t)), \delta(t)), \quad Y(t) \in \mathcal{Y}, \qquad (2)$$

where $\mathcal{G}(t, Y(t), Y(t - \tau), \delta) = \mathcal{F}(t, Y(t), u(t, Y(t - \tau)), \delta)$.

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$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t}\gamma_2(|Y|_{[t_0 - \bar{\tau}, t_0]})\right) \qquad \text{(UGAS)}$$

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$$|Y(t)| \leq \gamma_1 \left(e^{t_0 - t} \gamma_2(|Y|_{[t_0 - \bar{\tau}, t_0]})\right) + \gamma_3(|\delta|_{[t_0, t]}) \qquad \text{(ISS)}$$

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Without explicit flow maps, prove UGAS and ISS indirectly.

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O. Bernard, D. Dochain, J. Gouze, J. Monod, H. Smith, ...

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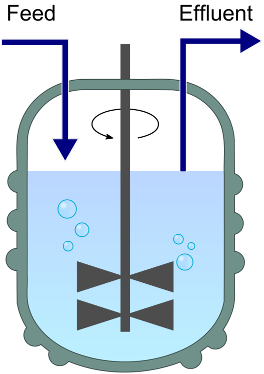

Constant volume. Substrate pumped in and substrate/biomass mixture pumped out at same rate

Uncertain Controlled Chemostat with Sampling

$$\begin{cases} \dot{s}(t) = D(s(t-\tau(t))[s_{\text{in}} - s(t)] - (1+\delta(t))\mu(s(t))x(t) \\ \dot{x}(t) = [(1+\delta(t))\mu(s(t)) - D(s(t-\tau(t))]x(t) \end{cases} \tag{C}$$

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$0 < \epsilon_1 \leq t_{j+1} - t_j \leq \epsilon_2$. $\delta : [0, \infty) \to [\underline{d}, \infty)$, with $\underline{d} \in (-1, 0]$.

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Assumption 1: The function μ is C^1 and $\mu(0) = 0$. Also, there is a constant $s_M \in (0, s_{\rm in}]$ such that $\mu'(s) > 0$ for all $s \in [0, s_M)$ and $\mu'(s) \leq 0$ for all $s \in [s_M, \infty)$. Finally, $\mu(s) > 0$ for all $s > 0$.

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$\mu(s) \overset{(\star)}{=} \frac{\mu_1(s)}{1+\gamma(s)}$, with a unique maximizer $s_M \in (0, s_{\text{in}}]$

Lemma: Under Assumption 1, there are $\mu_1 \in C^1 \cap \mathcal{K}_\infty$ and a nondecreasing C^1 function $\gamma : \mathbb{R} \to [0, \infty)$ such that $(\star)$ holds for all $s \geq 0$, $\mu_1'(s) > 0$ on $[0, \infty)$, and $\gamma'(s) > 0$ on $[s_M, \infty)$.

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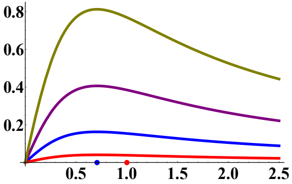

$(1+\delta)\mu(s)$ for Different Constant δ Choices, $s_M = 1/\sqrt{2}$ and $s_{\rm in} = 1$

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Goal: Under suitable conditions on an upper bound $\bar{\tau}$ for the delay $\tau(t)$, and for constants $s_* \in (0, s_{\text{in}})$, design the control D to render the dynamics for $Y(t) = (s(t), x(t)) - (s_*, s_{\text{in}} - s_*)$ ISS.

One of Our Results for Unperturbed Case

$$\omega_s = \inf_{s\in[0,s_{\rm in}]} \mu_1'(s)\ ,\ \ \omega_l = \sup_{s\in[0,s_{\rm in}]} \mu_1'(s)\ ,\ \ \rho_l = \sup_{s\in[0,s_{\rm in}]} \gamma'(s),$$

$$\rho_m = \frac{\rho_l^2}{2\omega_s} \max_{l\in[0,s_{\rm in}]} \frac{\mu_1^2(l+1.1\mu_1(s_*)s_{\rm in}\bar{\tau})}{1+\gamma(l)},\ \ \text{where}\ \ \mu(s) = \frac{\mu_1(s)}{1+\gamma(s)}$$

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Assume that $\frac{\mu_1(s_{\rm in})}{1+\gamma(s_{\rm in})} - \frac{\mu_1(s_*)}{1+\gamma(s_{\rm in}-\mu_1(s_*)s_{\rm in}\bar{\tau})} \overset{(a)}{>} 0$

and $\bar{\tau} \overset{(b)}{<} \max\left\{\frac{1}{2s_{\rm in}\sqrt{2\rho_m\omega_l}}, \frac{1}{2\rho_l s_{\rm in}\mu_1(s_{\rm in})}\right\}$, with $s_* < s_{\rm in}$.

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Theorem 1: For all componentwise positive initial conditions, all solutions $(s, x)(t)$ of the chemostat system (C) with $\delta(t) = 0$ and

$$D(s(t-\tau(t))) = \frac{\mu_1(s_*)}{1+\gamma(s(t-\tau(t)))} \tag{3}$$

remain in $(0, \infty)^2$ and converge to $(s_*, s_{\rm in} - s_*)$ as $t \to +\infty$. □

Extensions and Ideas of Proof

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ISS with respect to $\delta(t)$ without upper bounds on $|\delta|_\infty$...

$$\frac{(1+\underline{d})\mu_1(s_{\text{in}})}{1+\gamma(s_{\text{in}})} - \frac{\mu_1(s_*)}{1+\gamma(s_{\text{in}}-\mu_1(s_*)s_{\text{in}}\bar{\tau})} > 0 \quad ...(1+\delta(t))\mu(s(t))...$$

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$$\mathcal{U}_2(s_t) = \int_0^{s(t)-s_*} \frac{m}{s_{\text{in}}-s_*-m}\mathrm{d}m + 2\rho_m\bar{\tau}\int_{t-\bar{\tau}}^{t}\int_{\ell}^{t}(\dot{s}(m))^2\mathrm{d}m\,\mathrm{d}\ell.$$

Use $z = s_{\text{in}} - s - x = (s_{\text{in}} - s_* - x) + (s_* - s) \to 0$ exponentially.

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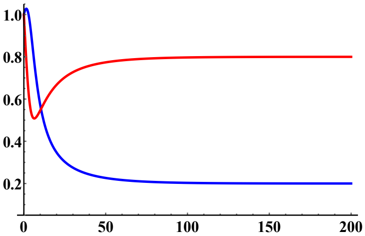
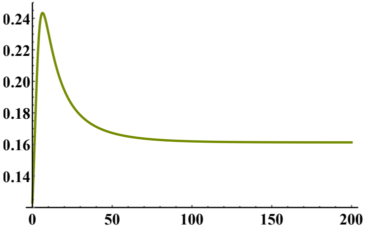
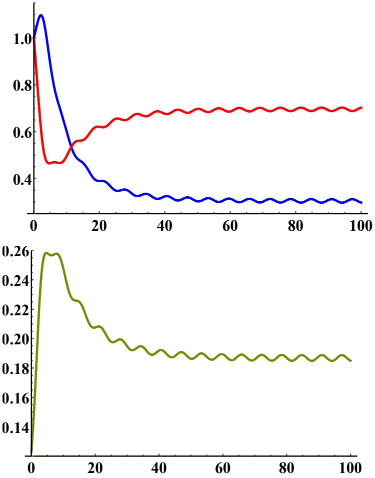
Mazenc, F., J. Harmand, and M. Malisoff, "Stabilization in a chemostat with sampled and delayed measurements and uncertain growth functions," *Automatica*, 78:241-249, 2017.





$s_{\text{in}} = 1, \; \mu(s) = \frac{0.5s}{1+0.25s+2s^2}, \; t_j = 0.24j, \; \delta(t) = 0.$

$s(t)$ in Red, $x(t)$ in Blue, $D(t)$ in Green.



$$s_{\text{in}} = 1,\ \mu(s) = \frac{0.5s}{1+0.25s+2s^2},\ t_j = 0.24j,\ \delta(t) = 0.15(1+\sin(t)).$$

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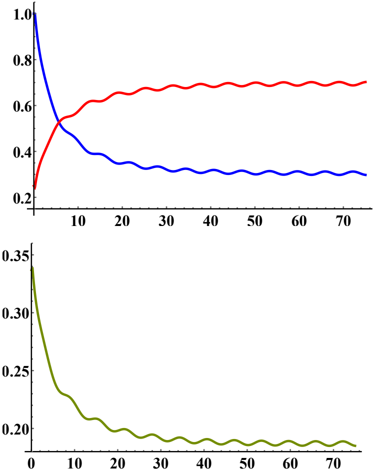

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Thank you for your attention!