Adaptive Tracking and Parameter Identification for Nonlinear Control Systems

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Sponsored by AFOSR, NSF/DMS, and NSF/ECCS

Department of Mathematical Sciences Talk
UT Dallas – February 2012

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where $\mathcal{G}(t, Y, d) = \mathcal{F}(t, Y, u(t, Y), d)$.

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Find γ_i's by building certain strict LFs for $\dot{Y} = \mathcal{G}(t, Y, 0)$.

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Warning 1: For each constant $\bar{\delta} > 0$, we can find a Y_0 such that the solution $\phi(t, Y_0)$ for $\dot{Y} = -\frac{Y}{1+Y^2} + \bar{\delta}$ is unbounded.

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$$\dot{Y} = f(t, Y) + g(t, Y)\Big[u(t, Y) - \overbrace{D_x V(t, Y) \cdot g(t, Y)}^{L_g V(t,Y)} + \delta\Big] \tag{4}$$

is ISS with respect to actuator errors δ in any control set.

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$(x, z) \in \mathbb{R}^{r+s}$. $\Gamma = (\theta, \psi) = (\theta_1, ..., \theta_s, \psi_1, \ldots, \psi_s) \in \mathbb{R}^{p_1+\ldots+p_s+s}$.

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Main PE Assumption: positive definiteness of the matrices

$$\mathcal{P}_i \stackrel{\text{def}}{=} \int_0^T \lambda_i^\top(t)\lambda_i(t)\,\mathrm{d}t, \;\; 1 \leq i \leq s \tag{10}$$

where $\lambda_i(t) = (k_i(\xi_R(t)), \dot{z}_{R,i}(t) - g_i(\xi_R(t)))$ for each i.

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$$\underline{\psi} < \psi_i < \overline{\psi} \quad \text{and} \quad |\theta_i| < \theta_M \tag{12}$$

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The estimator and feedback can only depend on things we know.

Augmented Error Dynamics

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$$\left\{\begin{array}{rcl}\dot{\tilde{x}} & = & f(\tilde{\xi}+\xi_R(t)) - f(\xi_R(t)) \\ \dot{\tilde{z}}_i & = & v_{f,i}(t,\tilde{\xi}) + k_i(\tilde{\xi}+\xi_R(t))\cdot\widetilde{\theta}_i \\ & & +\widetilde{\psi}_i u_i(t,\tilde{\xi},\hat{\theta},\hat{\psi}),\ \ 1\le i\le s \\ \dot{\widetilde{\theta}}_{i,j} & = & -\left(\hat{\theta}^2_{i,j}-\theta^2_M\right)\varpi_{i,j},\ 1\le i\le s\ ,1\le j\le p_i \\ \dot{\widetilde{\psi}}_i & = & -\left(\hat{\psi}_i-\underline{\psi}\right)\left(\hat{\psi}_i-\overline{\psi}\right)\Upsilon_i,\ 1\le i\le s\ . \end{array}\right. \tag{16}$$

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Tracking error: $\tilde{\xi} = (\tilde{x}, \tilde{z}) = \xi - \xi_R = (x - x_R, z - z_R)$

Parameter estimation errors: $\widetilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\widetilde{\psi}_i = \psi_i - \hat{\psi}_i$

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$$\begin{array}{rcl}\mathcal{Y} & = & \mathbb{R}^{r+s}\times\left(\prod_{i=1}^{s}\left\{\prod_{j=1}^{p_i}(\theta_{i,j}-\theta_M,\theta_{i,j}+\theta_M)\right\}\right) \\ & & \times\left(\prod_{i=1}^{s}\left(\psi_i-\overline{\psi},\psi_i-\underline{\psi}\right)\right).\end{array}$$

Stabilization Analysis

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We build a strict LF for the augmented tracking and identification vector $Y = (\tilde{\xi}, \tilde{\theta}, \tilde{\psi}) = (\xi - \xi_R, \theta - \hat{\theta}, \psi - \hat{\psi})$ dynamics on $\mathcal{Y}$.

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We start with this nonstrict barrier type LF on $\mathcal{Y}$:

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\begin{aligned}
V_1(t, \tilde{\xi}, \tilde{\theta}, \tilde{\psi}) \;=\; & V(t, \tilde{\xi}) + \sum_{i=1}^{s} \sum_{j=1}^{p_i} \int_0^{\widetilde{\theta}_{i,j}} \frac{m}{\theta_M^2 - (m - \theta_{i,j})^2} \mathrm{d}m \\
& + \sum_{i=1}^{s} \int_0^{\widetilde{\psi}_i} \frac{m}{(\psi_i - m - \underline{\psi})(\overline{\psi} - \psi_i + m)} \mathrm{d}m\,.
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On $\mathcal{Y}$, $\dot{V}_1 \leq -W(\tilde{\xi})$ for some positive definite function W.

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We transform V_1 into the desired strict LF.

Our Transformation

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Theorem: We can construct $K \in \mathcal{K}_\infty \cap C^1$ such that

$$V^\sharp(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) \stackrel{\text{def}}{=} K\big(V_1(t,\tilde{\xi},\tilde{\theta},\tilde{\psi})\big) + \sum_{i=1}^{s} \overline{\Omega}_i(t,\tilde{\xi},\widetilde{\theta},\widetilde{\psi}) \ , \tag{17}$$

$$\text{where} \quad \overline{\Omega}_i(t,\tilde{\xi},\tilde{\theta},\tilde{\psi}) = \begin{aligned}[t] & -\tilde{z}_i\lambda_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \\ & +\frac{1}{T\overline{\psi}}\alpha_i^\top(\widetilde{\theta}_i,\widetilde{\psi}_i)\Omega_i(t)\alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) \ , \end{aligned} \tag{18}$$

$$\begin{aligned} \alpha_i(\widetilde{\theta}_i,\widetilde{\psi}_i) &= \begin{bmatrix} \tilde{\theta}_i\psi_i - \theta_i\tilde{\psi}_i \\ \widetilde{\psi}_i \end{bmatrix}, \quad \text{and} \\ \Omega_i(t) &= \int_{t-T}^{t}\int_{m}^{t}\lambda_i^\top(s)\lambda_i(s)\mathrm{d}s\,\mathrm{d}m \ , \end{aligned} \tag{19}$$

is a strict LF for the $Y = (\tilde{\xi},\tilde{\theta},\tilde{\psi})$ dynamics on $\mathcal{Y}$.

Application: BLDC Motor (Dawson-Hu-Burg)

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Linear magnetic circuit.

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Linear magnetic circuit. Drives single-link, direct-drive robot arm.

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- The unknown vectors $\beta_1 \in \mathbb{R}^2$ and $\beta_2 \in \mathbb{R}^3$ and unknown scalars γ_1 and γ_2 are the motor electric parameters.

Application: Marine Robots (with Georgia Tech)

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20 days of field work off Grand Isle.

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Curve Tracking Dynamics

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Robustly Forwardly Invariant Hexagons

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We can use U_2 to prove ISS of the perturbed closed loop system

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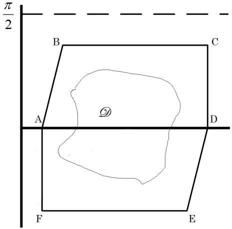

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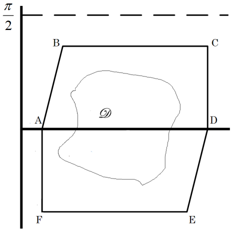

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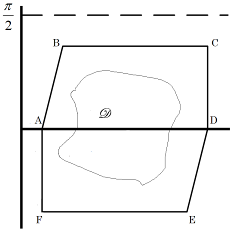

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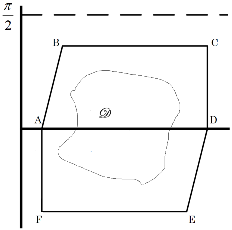

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We proved ISS of the tracking and identification dynamics

$$\begin{cases} \dot{\tilde{q}}_1 &= -\sin(\tilde{q}_2) \\ \dot{\tilde{q}}_2 &= \frac{\kappa\cos(\tilde{q}_2)}{1+\kappa(\tilde{q}_1+\rho_0)} - \frac{K_2}{\tilde{K}_2+K_2}u_2 - K_2\delta \\ \dot{\tilde{K}}_2 &= -(\tilde{K}_2+K_2-c_{\min})(c_{\max}-\tilde{K}_2-K_2)\frac{\partial U_2}{\partial\phi}\frac{u_2}{\tilde{K}_2+K_2} \end{cases} \quad (28)$$

for $(\tilde{q}_1, \tilde{q}_2, \tilde{K}_2) = (\rho-\rho_0, \phi, \hat{K}_2 - K_2)$ on each set in a nested sequence of sets that fill the state space.

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- We aim to find extensions that apply under time delays and state constraints including obstacle avoidance.