Calculus I and II with *Mathematica*: an Experiment at Louisiana State University

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Introduction

Last Fall we used *Mathematica* extensively in the teaching of two of LSU's 18 sections of Calculus I. Each of the authors met his 15 students for two hours per week of lecture and three hours per week devoted to student use of *Mathematica* notebooks in the department's one, small, 9-terminal NeXT lab. In the Spring of 1992, the first author taught a continuation of this experiment to a Calculus II class of 21, 14 of whom had taken an ordinary section of Calculus I in the Fall.

> We required the students to buy the Illinois text, *CALCULUS & Mathematica*, so that they had a printed copy of the electronic notebooks which were operating on our computer system. No additional text was recommended or required. At mid-year we installed *Mathematica 2.0* and added the new Vectors chapter, of which we used sections 5.01 and 5.02.
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> Observing our students' interactions with *Mathematica*, we discovered a number of unsolved difficulties, perhaps largely specific to our setting, in the nature and use of the electronic notebooks. In Calculus I we mostly just selected and assigned material and problems from the Illinois electronic text. In Calculus II, as we went along we revised, re-organized, and supplemented the Illinois materials extensively. Our purpose here is to describe and discuss the course design issues that seem important to us at this point. Our students' evaluations of the course are in the Appendices.

We find that the idea of teaching Calculus with *Mathematica* in a central and extensive role is interesting and promising. It seems clear that we are in only the early stages of developing materials and procedures to make the idea workable and satisfactory. We are confident that Calculus II can be taught successfully at our University in this new style. Our minds are open about Calculus I.

The next decade may bring helpful changes in the technology and in computer costs and accessibility, opening up new possibilities for course-wide and curriculum-wide transformation. Nevertheless we currently make our decisions in a setting of scarce resources, considering one course at a time. In the academic year 1992-93, we plan again to offer one *Mathematica* section of Calculus I in the Fall, and one of Calculus II in the Spring.

The other uses of our NeXT lab in 1992-93 will be to teach a new Fourier Analysis course inspired by Professor David Kammler's work at SIU in Carbondale, and a section or two of Calculus III. Both of those courses involve the computer rather less extensively. The Calculus III program has been under way, with pleasing results, since Spring 1991.

The conditions and circumstances which are relevant to an experiment like the one with Calculus I and II may differ in important ways from school to school. The particular conditions that hold at LSU of course shape our evaluation and conclusions. Therefore we must begin by discussing relevant characteristics of our students.

The Students

LSU freshmen who take calculus have an average ACT Mathematics Subscore of 27 (equivalent to a 540 QSAT). All have a certain competence in high school algebra and trigonometry, assured by the LSU placement test. Seventy-six percent of them pass Calculus I on the first try; 8% fail, 16% withdraw.

Almost none of them have taken any calculus before. Furthermore, there are areas of precalculus mathematics to which they have been introduced, but in which we customarily accept the need, in the calculus course, to refresh their minds and further develop their proficiency. Examples: They are not very familiar or handy with the elementary properties of the exponential function and the logarithm; or with important trigonometric identities like those for Cos[2x] and Sin[2x]. They are not so adept as we would wish at certain algebraic operations, like those done by *Mathematica's* commands Apart and Together.

Almost all of LSU's calculus freshmen have had a semester or more of computer studies in high school. But at least at first, most of those in our Calculus I found the discipline of using *Mathematica*, and of dealing with a computer system, daunting and distracting. A significant number decided that the computer's involvement was too much of an additional burden, and they fled to one of the sixteen other sections, which were being taught in classes of 40 using Earl W. Swokowski's text (Calculus, 5th edition, PWS-Kent, Boston, 1991). There was enough emigration to be disruptive.

The question becomes, Are the students sufficiently impressed by what they are gaining so that they find the added stress and strain to be tolerable? With Calculus I, the answer was too often No. With Calculus II there was much less enrollment flux, and none after the first three weeks, apparently because the students knew enough calculus to appreciate, from the outset, what *Mathematica* could do for them. This maturity effect is all the more pronounced in Calculus III, where students embrace readily the opportunity to use *Mathematica* for their homework.

Non-academic characteristics of our students' lives are also important. Compared with students at U.C. Berkeley, or U.I. Champagne-Urbana, LSU students are less likely to be financially secure, full-time, resident students. They often work 20 hours a week, and they count on being able to keep the time demands of their academic courses within predicted bounds. Their schedules often make it difficult to put in even the basic extra hours for homework in the lab, or for team members to find common times for working together.

Not a Scientific Experiment

We will offer some judgments and advice out of our experience with this project. But as we had to realize from the beginning, this could not be a scientific experiment. It was not a matter of comparing the success of two methods, differing only in a small number of well-defined ways, in achieving the same goals with two student populations whose characteristics were the same.

We were altering both the objectives and the methods. We were at once re-inventing the syllabus and trying a new teaching program. We were constructing a course around the strengths of the computer setting. In the tests and assignments, we were measuring somewhat different achievements and skills than in the usual calculus.

It was not practical to select our students or establish a control group. We could only try to observe and be thoughtful about the characteristics of those who selected us, and of those who started the course and then fled.

Then too, our project had important features which were quite independent of the use of the computer, even though conditioned or occasioned by it. To wit: There was a substantial weekly problem set, which was required and carefully graded. We emphasized writing somewhat more than usual. The students did the problem sets in teams of 2 or 3 students. The student-faculty ratio was much lower than usual. We gave more hour tests than usual. Give that much additional attention to any population of our students, with or without computers, and you'd surely see significant differences in the outcome!

Finally, it remains to evaluate the effects on these students' future studies, both in the immediate future, as they enter conventional sections of Calculus N+1, and in the long run as they find, or do not find, that computer calculus has given them something of value.

The Objectives and Content of Calculus

The Illinois materials are of great value. They represent an elaborated proposal of substantial change in the definition of the course. Whether we find them successful or not, they shake up and re-start our thinking about The Great American Calculus Course -- that long-settled compromise responding, we all presume, to the needs of the engineering colleges; that lengthy syllabus which we calculus teachers often feel is overstuffed, overreaching, and barely cohering. The

Illinois writers lead us to ask not merely what kinds of knowledge and technique we want to impart (in preparation for Calculus N+1), but also what kinds of abilities we *should* be trying to develop. They lead us to think critically about what the students most need, and what we can really teach effectively.

These impulses and questions are not altogether new. We have been de-emphasizing computational virtuosity, in finding anti-derivatives for example, for some years, in view of the computer's presence in the world beyond calculus -- hasn't everybody? Haven't we all been adjusting our courses toward greater coherence and greater realism? No?

In any event, it appears to us that changes in the objectives and content of The Great American Calculus Course will arrive in evolutionary fashion. We shall have to argue for them, test and prove them, one at a time.

The key new element in our experiment is the use of *Mathematica*. If the presence of such a tool in the student's daily lives, *per se*, leads to changes in the content of Calculus, it will be because certain topics become easier to teach with that tool, or because some things become practical to teach which were not practical without it. To be sure, we do not propose excluding material just because it's the hardest to teach, or including it just because it becomes fun; but the practicality of a goal at a given stage is always a legitimate consideration in course design. So we shall undertake now to say what *Mathematica* is good for, or might be good for when we become smarter at using it.

The Uses of *Mathematica*

By conducting the course as we did, we were testing some very ambitious claims for the activities involving *Mathematica*, which we may state as follows:

(1) *Mathematica* notebooks can completely replace a conventional printed text.

(2) The students' interaction with *Mathematica* notebooks and on-computer projects and problem sets can largely displace blackboard lectures as the primary source of instruction.

(3) To a large extent, the evaluation of students' work on *Mathematica* projects and problem sets can replace evaluation by conventional on-paper testing.

(4) Having the students work in teams is educationally beneficial in itself.

(5) Having at hand the powers of *Mathematica* allows a redefinition of the course's content and objectives.

We will discuss those claims one by one.

Claim 1: *Mathematica* notebooks can completely replace a conventional printed text. Of course,any printed text can be displayed on-screen. At best, there is no gain from simply doing that. The question here may be stated as follows. Which subject matter can benefit from skillful incorporation into an electronic notebook? Or, which parts of the study of a subject can benefit?

An electronic text introduces a topic, carries out some preliminaries, and presents a few examples, perhaps by presenting output from some *Mathematica* commands. Then it draws the student into an interactive mode; it suggests that he or she try variations on the examples by modifying and re-executing the *Mathematica* commands; then, with luck, induces a period of still more active and playful investigation; and finally leads the student into wonderfully instructive homework to be done with *Mathematica*. That's the idea, and it's a worthy one. Let us point out some limitations and pitfalls.

An electronic text that is trying to be the only text will, in parts, be performing the same functions as a traditional printed text. For example, at introductory stages it may give definitions, identify the topic, state results, describe methods, name applications, make historical remarks, and present examples. At other times, it must serve as a reference, so that the student who needs review of an earlier topic can approach it through an index or table of contents and find a definition, fact, technique, example, or theorem. For the most part, when writing electronic text to serve those purposes, one does not need to engage the powers of the computer beyond those of displaying print and pictures on-screen. Those purposes are just as well served on the printed page -- even were it not the case that time at a terminal is a scarce resource accessible only at certain hours and locations.

Moreover, the value of the interactive mode is variable. Its teaching potential depends on the nature and stage of the subject matter in question. At the beginning of a topic, we may have the opportunity to impart a basic concept by drawing the student through the on-paper solution of simple cases. Merely by engaging the computer too soon, we throw away that opportunity. Entered too soon, the interactive mode may leave the student excessively dependent on the machine.

There's no harm, and in fact it seems to us the most natural and convenient thing, if electronic text is the companion, rather than the replacement, of a printed text - it being understood that notation and organization should be compatible. The writer of an electronic text will do a better job if he or she is constrained to start at a point in the subject matter where the capabilities of *Mathematica* are clearly needed and worth bothering with, and then to get right down to the business of inducing student interaction and experimentation. Certainly it is unhelpful if one works under an obligation to weave *Mathematica* commands into every stage of every topic, as if we couldn't even differentiate x without it.

We became convinced of *Mathematica* notebooks' potentially substantial value in, for example, the following areas:

(a) *Mathematica* let the students compare the uses of interpolating polynomials and least-square-degree-K approximations in dealing with a function known only at N points; it gave them easy and quick access to many examples, not limited to small N . Without the computer to do that, we would probably not propose to mention those topics in freshman calculus. To assure that they grasped the two concepts, we found it natural to teach them how to compute the objects on paper for the cases N=3 . That was easier to do away from the computer.

(b) Thanks to the computer's ability to do tedious calculations and plots quickly, *Mathematica* made it practical to teach "data analysis," as an application of the elementary properties of the logarithm and exponential functions: Begin with the plot of N data points (a,b), pass as indicated to the plot of the points (a,Log[b]) or (Log[a],Log[b]) to find an apparently linear relation, use a least-squares linear approximant, then write down a guess at the original function. However, we think it hinders students' ability to handle such problems if the electronic text writer is in the habit of using *Mathematica* commands to pass (for example) from Log[f[x]] = 2*Log[x] to f[x] = x^2. Unless your class is pre-educated and beyond harm in such matters, we advise having the computer turned off while teaching easy, basic concepts for which the tool of choice is a pencil, or a thought.

(c) Thanks to its graphics capability, *Mathematica* can be brilliantly useful in teaching the geometry of space and of vector-valued functions. The student can, in the ParameterPlot command, replace the parameter domain of a parametrized curve or surface by subsets thereof, get different parts of the object on screen, and figure out how the parametrization works. It becomes easy to learn why different parametrizations of the same path entail differently-behaving tangent and acceleration vectors. The properties of the cross product are easy to illustrate in electronic text, and students can be led through the discovery of its algebraic properties. On the other hand, we recommend teaching the students to handle the basics of points, lines, and planes in space before turning on the computer. If *Mathematica* does even the trivial things for them from the beginning, they may come away from the course without the basic concepts, or with a competence that will work for them only in the precise setting of the course.

We could list several other areas in which we are enthusiastic. But does every step in Calculus need to be presented within an electronic text? Is every topic suited to be taught that way? We think the answer is No.

Claim 2: The students' interaction with *Mathematica* notebooks and on-computer projects and problem sets can largely displace blackboard lectures as the primary source of instruction. We agree. As we all know, the important thing is to get students to be active. Yes, it's possible that they will listen actively to a lecture, read the printed text actively, do exercises on their own, and study thoughtfully. They often do. But we are all in search of techniques to get them to be active more often and more consistently. The electronic notebooks and the on-computer homework problems appear to do the trick. We find it much more fun than giving the usual lecture when we walk into the lab, find that the students have arrived early, see them experiment with the computer, hear them talk things over with each other and ask us good questions, and watch them make discoveries.

But lest there be any doubt: When we replace an hour of lecture with an hour of lab, we are not dispensing with an hour of the teacher's time. The students need the teacher in the lab to get them past the occasional orneriness of *Mathematica* commands, to answer their questions, to talk over their discoveries, and to clear up confusion. That makes the course all the more expensive to teach, if the lab is small and can serve only so many students at a time.

Claim 3: To a large extent, the evaluation of students' work on *Mathematica* projects and problem sets can replace evaluation by conventional on-paper testing. We agree. Last year we counted the on-computer assignments 40% of the course grade. There are some difficulties, but perhaps they are not insurmountable. If the on-computer projects and problems are to be a defining activity of the course and determine much of the students' grade, then of course we need to put in more time developing and improving the validity and quality of those activities. Then, too, time and resources are short, both for getting the electronic "papers" graded in a timely and helpful fashion, and for providing the students with all the computer time they need to work individually. --Which brings us to the next Claim.

Claim 4: Having the students work in teams is educationally beneficial in itself. Let the students speak on this one; see the Calculus II students' responses to Questions 6, 7, and 8 in Appendix 1.

Claim 5: Having at hand the powers of *Mathematica* allows a redefinition of the course's content and objectives. We agree. Let us count the ways, and state some reservations.

1. *Mathematica* will let us pursue concepts into realms formerly prohibited by their computational difficulty. We can go from the set-up of the definite integral to the numerical answer in seconds! So we can be spared the development of computational virtuosity, and our students can also, even so, work with substantial and interesting examples and applications. See the forest better, and more trees too.

Need we say it? --Let's not get silly. The school child's $5 calculator will keep the addition of ten 5-digit numbers from being an obtrusive part of her life, and that's all very well. But the process shouldn't be a mystery; she needs to know enough arithmetic to tell when her battery's dead; and there should be a limit to how helpless she will be if it is. Just so, our engineering student's ability to find an antiderivative by hand should carry her some distance along the list

x, x^p, Cos[x], Cos[x]^2, x Cos[x^2], 1/(1+x^2), u'[x]/u[x], . . . ;

perhaps she should know how, in a pinch, to integrate any rational expression over the reals. Wherever we draw the line, we still have the job of teaching that much integration technique.

2. *Mathematica* may open up a new and better manner of learning, through inducing the student's experimenting and discovering things. Let the student experiment with the partial sums of the harmonic series and its variants, and thereby learn how partial sums of infinite series can behave; *then* discuss the general principles. Let the student experiment with how partial sums of a Taylor series approximate the function -- closely on compact subintervals of the set of convergence, perhaps not so well near the endpoints; *later,* define the radius of convergence, uniform convergence,and such. Such pedagogical ideas have to be explored and tested one at a time for practicality.

3. Through the combination of its advantages, *Mathematica* may lengthen the list of subjects which it is practical to teach in Calculus I and II. Let us offer an example from our experience in Calculus II. *Mathematica* made it possible, for the first time in our experience, to present successfully the derivation of Kepler's Laws, point out their position in the history of science, and elaborate a bit on their meaning. It is an odd example to offer, in a way, because we dealt with the Laws only in the last three blackboard lectures of the course, and they were absent from the computer materials. Furthermore, the texture of this material - a logical derivation within the mathematical model provided by the Law of Gravitation and the Second Law of Motion - is unlike anything else in freshman calculus. Yet we think these lectures were accessible and understandable, and they seemed to be appreciated, particularly by the physics major in the class; and that's all we wanted.

The point is that *Mathematica* had let us prepare the way, so that at each line of the argument we could expect the geometric and physical meaning to shine through the formulas. We are convinced that *Mathematica* makes it easier to teach and learn vectors, points/lines/planes/distances in space, the dot and cross product, parametrized space curves, tangent and acceleration vectors, curvature, torsion, and the moving frame T/N/B on a space curve. Bit by bit throughout the semester the students had been gaining a sense of these things by visualizing examples on-screen. Without *Mathematica,* in some of this material, the formulas and computations will slow you down and defeat you. With it, the symbolic manipulations and the computations are done in seconds; they are no longer obtrusive, but slip into the background.

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It remains to be seen whether *Mathematica* will allow us, in time, to teach a fundamentally better calculus course. We think it may. The Appendices below contain the opinions of our students. They found it exciting and pleasing to be part of the experiment, and they have encouraged us to persist, which we will. The world waits for the economics of it to become workable, and for excellent materials and methods to be developed. Eschew overstatement. Lay one brick at a time.

Appendix 1

Student Opinions of Calculus II with *Mathematica*

Eight of the 21 students in the Calculus II class responded to a special questionnaire, and their responses are collected here in their entirety, unexpurgated. There is one item of strong personal praise for the instructor. We considered deleting it, but we believe our readers know that such statements indirectly measure satisfaction with the course as a whole; there is much else to balance it off; and besides, we enjoy reading it.

Under each question, all the students' responses to it are grouped together. The students are identified by their initials, and to satisfy the reader's curiosity we list their course grades:

RB: B--	KC: B+	SG: B	AH: A--
TM: A--	CO: C+	MS: C++	JS: B

AH was a senior completing a degree in Quantitative Business Analysis and planning to earn a second degree in Mathematics. He had also taken Calculus I with *Mathematica*. He was a very conscientious and hard-working student, and had to work especially hard to adjust to something other than the rote, pat learning procedures to which he was accustomed.

CO is a senior in geology, satisfying his calculus requirement late in his career.

MS is a physics major, and was making an A in the course until the last six weeks, when other interests diverted him.

JS is a graduate student in philosophy.

Question 1. A mathematics course usually involves reading a text printed on paper, perhaps using a pencil to take notes and work things out as you go along. To a large extent, our course replaces that activity with the reading of *Mathematica* files on screen, using *Mathematica* commands to try things out and interact with the text. In your experience, what's the difference? What advantages do you see in one or the other?

RB: The biggest difference I see is in the actual working of the problems. M. takes out much of the dirty work in solving long problems where careless errors usually occur. It is also nice to have the 3-D graphing capabilities at hand when working with those functions parametrized in R^3. It helps with the visualization process involved with such curves and surfaces by its capability to produce a 3-D graph in just a few seconds.

KC: I see the advantage of being able to see the logic and reasoning done without all of the hard work needed to be computed. This allows me to concentrate more on the problem than on how to work on the problem. It is much better than seeing it in a book and not being able to play around with different alternatives to given problems - like changing a variable to see what happens, which a book won't do for you.

SG: In taking a course such as this, the student has a better chance to visualize the material that is presented to him, since it can be drawn by the computer with better speed and accuracy than the student can. However, if the student is in the pen-and-pencil version of the 1552 course, it appears that the student would gain greater practice of developing the skills that the computer performs. The student is more dependent on himself, and thus seems to understand more of the material.

AH: Personally, I was never really a big fan of reading text on the computer screen. So, perhaps I am a bit prejudiced in answering this question.

First, I feel that it is necessary to have a textbook which will serve both as a reference and as a study guide. This textbook should be thorough and complete in itself. It should have plenty of examples worked out for the student in a well-presented, easy-to-understand format. Also, I feel that there should be a reasonable number of exercises presented at the end of each section, with the answers worked out in the back of book. I think that this was a serious omission in the present format of the course. Believe it or not, answers help. And answers which are worked out for the student can actually help to clear the fog which surrounds some fundamental concepts .

To be sure, we should have had much more drill in the "pencil and paper" methods of taking derivatives, integrals, working with vectors, and general mechanics of Calculus .

I feel that the lab work should consist only of problem sets with the minimum instruction necessary to introduce the topic and get the student going. NO ONE is going to take the time to go through the trouble of reading the Calculus & *Mathematica* text on screen. So it is a waste of time to include the full text on computer. Indeed, very little lab time is already available to the student, and the student will naturally maximize his available time by using the lab to solve problems he will be graded on.

TM: A calculus text such as the one by Swokowski generally has more vivid examples and deals more on the method to solve given problems only, whereas *Mathematica* deals with more theory and a wider range of applications to the subject material. Another difference, it is easier to look up a specific problem in a textbook, and it is easier to study from a textbook like Swokowski that is clear and to the point. Some of the files rambled on about material that was trivial or irrelevant.

Mathematica does have a distinct advantage over a "dead" textbook in that it is much more interactive, and in the sense it is a "living" text, because it can offer unlimited and precise examples instead of a student contacting a teacher. But I find one drawback to the way our particular classwork was assigned. We never had time to really go through the files and experiment with them because we were in such a rush to finish our problem sets that were due every week, so I didn't really benefit from this advantage of *Mathematica*.

CO: It makes you have to read the text to do the homework, but it is not as informative as it could be and doesn't explain stuff very well.

MS: The advantages to this method is that much of the time in a normal Math 1552 course is used in making sheer calculations. Many teachers are of the opinion that practice makes perfect when it comes to integrals, but doing the same type of integral 3000 times is a little much. Once the student gets integration techniques for a new type of integral down, *Mathematica* is there to do the dirty work for them. Therefore, students can spend more time, in class and out, learning theory and application rather than merely computational skills. Also, no matter how good of a drafter a student is, there is no way that he will be able to get all the 3-space graphs right. *Mathematica* puts it out perfectly, so the student can see the right answer and the spatial relationships involved.

JS: An interactive *Mathematica* text holds the advantages that: 1) One is able to perform large calculations that would otherwise be prohibited because of time. 2) Some of the more abstract relations in calculus which can only be grasped through repeated analysis of sometimes complex graphs can be easily seen. 3) The understanding of procedure which has been the focus of mathematics education is subordinated to understanding of concepts - the end result being that the procedural knowledge is actually improved because students know why relationships between problems and the method of interpreting them work rather than to perform such and such a algorithm on this or that string of symbols.

However, there is I feel still a need for some sort of text complement for the course. In some ways I feel that not having a calculus text and the knowledge of how to use that particular text cheats me of one of the great benefits of taking a full year of calculus. All the people that I know who have taken calculus constantly refer to a text in a religious manner, and that is something that I will have to gain outside the scope of this class. Also I feel a need to have a formatted 'hard copy' available for studying the *Mathematica* files. I understand that the reasons that this is not yet available but would advocate a search for an adequate textual supplement for the course.

Question 2. A mathematics course usually involves doing homework on paper. Our course replaces that activity with doing homework on computer, supported by *Mathematica*. In your experience, what's the difference? What advantages do you see in one or the other?

RB: With this course, the most notable difference is the emphasis on the homework. According to the syllabus, homework counts for 40% of the final grade in the course. This forces students to actually DO the homework and think about it, which is not the case in any traditional Math classes at LSU. Also, in having to do the homework in the lab(since the University will not let us take the computers home), there are generally several students working on the same things in the lab at the same time. If everyone is having trouble on one problem, everyone can usually combine wits to come up with a usable process for finding the answer. We also give each other hints as we go along so that everyone can LEARN the process and not just get the right answer.

KC: I see no difference in doing the work on paper or computer. The advantage of paper is that you work on your own time at your discretion. With *Mathematica*, you work on set lab schedules which don't always fit a schedule.

SG: The student who does the homework with paper-and-pencil gains a great deal of practice from this method, while the student at the computer seems to spend more time going into all the problem sets trying to find the problems that are similar to the homework, so he can transfer those commands to be used in the homework. He seems to spend more time trying to learn the command language than the calculus, which is the purpose of the course.

AH: I feel that both approaches should be emphasized equally. For instance, there are certain calculus fundamentals that students must learn to do "pencil and paper." These important concepts should first be presented in class supplemented with "pencil and paper" homework. These concepts should then be enforced by presenting them on the computer in the form of *Mathematica* problem sets. I feel this method would greatly enhance the student's understanding of the material by focusing on both fundamental "pencil and paper" skills and then reinforcing these skills with the power of *Mathematica*. We need to understand that a student can fully appreciate the power of *Mathematica* only after first going though the pain and suffering of solving challenging calculus problems by hand.

So in summary, the computer should be used for three purposes:

a) As a powerful supplementary tool used to enhance a student's understanding of calculus in particular and mathematics in general.

b) As an powerful example of how much more effective and productive the student can become in his work when *Mathematica* is used to solve practical problems and enhance theoretical concepts.

c) As a pathway from the world of painstaking hand-computation to the modern world of efficient, computer-based problem solving.

TM: The homework on the computer would probably be very difficult to do by hand. It required much thought, calculation and overall effort. I don't think that I learned the basic methods of calculation, because instead of doing many easy problems, then building up to some more difficult problems, our problem sets were comprised of only medium and high-difficulty problems. This seems to be a major disadvantage in the long run, because the material that is retained the longest in a math class is that subject matter which is learned through repetition. Also, *Mathematica* did much if not all of the calculation for you; so you didn't have that added learning from actually doing the work by hand. So when we went in for the tests there was major difficulty in doing some things by hand.

CO: The one main problem I have had is the computer going down.

MS: One disadvantage to doing homework on the computer is the ease with which unscrupulous people can copy other people's homework. I've seen it done, and it really is easy. One way to solve this would be to change the groups of the file, allowing only the owner to read it. (In UNIX terms, "-rwx------") Plus: emphasize to the students the importance of changing the password when changing teams. Other than those concerns, homework on the computer is much better. The concepts can be more advanced because the instructor doesn't have to worry about giving the student too complicated of an integral. He can make it a page and a half long, if need be (and you have! :)) Plus, again, the graphs can really be visualized correctly on *Mathematica.*

JS: Again many of the same advantages and disadvantages apply. I think that this course would be greatly enhanced with some pencil and paper requirement before students could begin on the actual *Mathematica* notebooks. This may pose some time problem during the semester as we continue to add to the burden of the students who take *Mathematica* based calculus. These students however to this course to learn about both calculus and the use of *Mathematica* so they should be able to handle the extra work load (assuming that they did not take the course because it was the only open section, and assuming that the extra work clearly augments the understanding of calculus). Perhaps as the course is further developed a work at your own pace system, with some on-line test, and some pencil and paper test could be developed, students would then have to perform some rudimentary pencil and paper homework to be finished before they could work at the broader understanding provided by the notebooks.

Question 3. Math 1552 usually involves five hours a week of lecture. Our course replaces three of those hours with time on computer, reading *Mathematica* tutorial files to learn the material. What do you think about that change? Would you like to have more lectures and fewer lab hours, or fewer lectures and more lab hours? Or is the ratio 2:3 just right?

RB: I personally feel that this is probably the best ratio, since calculus lectures generally put me to sleep, and I can stay awake at a computer terminal for many hours longer than I could in those little brown desks in those little lifeless classrooms. The two hours of lecture usually are enough time for the teacher to go over important or difficult concepts which may not be covered enough in the electronic text. More computer time is better in my opinion than lectures.

KC: I would rather see an integration of lecture and computer. Maybe the first third of an hour dedicated to lecture and the last to working on the problem sets on computer. This way the information presented on a certain day could also be applied on that day on the homework which could be arranged so that the info is applied on that day.

SG: There should be more lecture time than computer time. The computer should be more a supplement to the course than the basis.

AH: Though I believe in the idea of using *Mathematica* to help students gain a greater appreciation for mathematics, I feel that adequate lecture time is absolutely necessary for students to be able to understand a subject as challenging as Calculus. I don't feel that two 50-minute sessions of lecture per week can adequately prepare the students for solving problems requiring advanced mathematical techniques.

What's more, the students aren't going to spend the time reading the text on the screen. Rather they will spend their available lab time working on problem sets that will be graded.

I think the way the course is organized needs to be reviewed and changes made. I think at least three lectures per week are necessary -- possibly four. And the lab time should be designated as lab, similar to the way chemistry, physics, and biology are organized. So you would still have a five hour course, but it would be divided into a three-hour course and a two-hour lab and would complete the sequence in three, rather than two semesters.

TM: I never got used to the 2 hours of lecture per week program. It seemed that the lecture rarely if ever correlated to the material we were working on in the lab. The lectures were so complicated that most of the people in the class "blew them off". An entire hour of just theory can get pretty long and boring. I would like to see more applications and examples and more diverse subjects covered instead of a few topics dwelled on at length.

As far as lab hours go, the time in the lab with Dr. McGehee was very beneficial in helping out with the problem sets. But often (always) the problem sets took more than three hours to complete, so we were in the lab with no one to explain or help us with the problem sets. I would like to see lab proctors who are knowledgeable on both the Calculus II material and *Mathematica.*

CO: I would rather have more labs but in the labs it would be better to have more instruction on the applications used in the homework.

MS: I think you ought to switch it to 3 hours of lecture and 2 hours of lab time. Most students come in after class hours anyway. But you can only get lectures during class periods.

JS: I feel that the present ratio is just right. It depends heavily on the format of the lectures. Last semester I felt that the lectures did not really help understand the material that I was covering independently with the notebooks and thought the lectures should be longer. This semester I felt that the highly organized lectures, and their pointed relationship to the weekly *Mathematica* assignments was quite appropriate. I might only suggest that the instructors provide a complete syllabus of the lecture topics and recommended reading for preparation of each lecture.

Question 4. On average, how many hours per week did you spend working in the NeXT Room (including the 3 scheduled class hours)?

RB: I spent 7-10 hours weekly in the NeXT room working on homework, reading the text files, and exploring the many capabilities of the NeXT computer such as VoiceMail,Composer, Poker, Billiards, Stealth.... The list goes on. If the homework gets too tedious, we can release some tension by playing a few hands at Poker, or a hopeless game of Chess against the computer.

KC: About 7-8.

SG: On the average, I spent about ten to twelve hours a week in the lab setting. I spent two or three Sundays in the lab as well. The majority of these times was during late morning and afternoon hours during the early part of the week.

AH: 13 to 17 hrs.

TM: On average, over 10 hours.

CO: 15.

MS: About 6.

JS: I would guess twelve to fifteen hours.

Question 5. On average, how many hours did you spend working on this course away from the computer?

RB: Not very many.

KC: 2 per week when we had lectures.

SG: About five hours a week, although maybe it was less.

AH: 4 to 5 hrs.

TM: 2 hours.

CO: 2.

MS: About 1 1/2.

JS: Five to ten hours

Question 6. When you did homework by teams, did the team members (1) divide up the assignment, so that each person did a part of it alone; or did you (2) work on the whole assignment together? Or which of those was more nearly true, on the whole?

RB: We just worked on them till they were done. If we were together, we worked together, if we were not here at the same time, we worked what we could when we were there.

KC: Each person did part of it alone. This was true in every case.

SG: My partner and I divided the homework assignments pretty evenly between ourselves. These we usually did a majority of the work alone, although we did constantly check each other's work, and made a point to let the other know what he had done. It is difficult for two people with very different schedules to get together and do the homework as a team.

AH: More often than not, our team divided homework assignments. Even with the shortened format, it was necessary to divide assignments so that an extraordinary amount of lab time was not required of each team member.

This had both advantages and disadvantages. The advantages were the time savings. The obvious disadvantage was the fact that sometimes a particular team member did not have a chance to work on a part of the problem set thus causing a gap in his knowledge and experience.

Even though teams who separated problem sets always promised themselves that they would go over and work the other partner's problems, that would usually never happen; there was just not enough time.

TM: On the first few problem sets my team members and I worked on the problem sets together. This was awkward and produced little progress until Wednesday night. Consequently I did most of the work, staying in the lab after hours very often to get the set in. Then I decided to split the set up, so that I could work at my own pace and get my problems done, and then help with my partner's problems. I thought this was a solution to the problem, but I still ended up doing most if not all of the problem sets.

CO: We worked on the homework mostly alone , but we did get together to do the work also about 1/4 of the time.

MS: Mostly we divided up the different parts, then checked over the other person's work.

JS: CN and I did the work together.

Question 7. Describe your particular, personal experiences, problems, and successes with doing homework on teams.

RB: I feel it was a valuable learning experience. When one team member doesn't understand a process, a fellow student can usually help them to understand by tutoring and questioning. Fellow students can often give a completely different perspective to working problems than someone with a doctorate in Math from a prestigious Ivy League school. (no offense)

KC: No good. Teamwork was barely existent in every case and only seemed to share grades among two or three people. You were lucky if you got to work with your partner at any time during the course on a regular schedule.

SG: The only problems we seemed to have would be if one of us were having a bad week test and/or otherwise, but the other usually was able to take up the slack of the other, so it evened out in the end.

AH: That is a relatively easy, albeit emotional question. Teamwork has tremendous advantages and disadvantages. No doubt, the old adage "two heads are better..." holds here. But that is only partially true.

If one is fortunate enough to have a good team member who is intelligent, industrious, resourceful,and prone to good work habits, then team work can be a joy and a very rewarding experience. Unfortunately, the majority of team members are usually none of the above. This is often due to "youthful indiscretion", but nevertheless is true. So, team work is usually a struggle, with a considerable amount of time and energy devoted to worrying about whether your partner will make the necessary commitment to the course and the problem sets and the grades.

For someone trying hard to make an A, a bad partner can spell disaster. For example, I had the advantage of having three good partners, DT, LF, and TM. All three of these individuals were committed to the course and to making a good grade as I was. The result was: I received an A on all problem sets with the exception of one where I received a B. But my last partner was a lemon and it had a very negative effect on my grade. What was really bad was that I was helpless to do anything about it.

But that's life working in teams. One just has to pray for a good team member.

TM: As in the previous answer, I endured much difficulty and stress over the homework problem sets. I nearly always ended up doing most of the work, and staying late nights in the lab. I did learn much from the lab work; but most of what I learned was patience, time management, and personal relations -- not calculus.

CO: It was very good working together in teams because it allowed us to work out problems as a team with one person lending a hand in stuff the other couldn't do.

MS: Well, some partners I did not get along with at all, and some partners I got along with very well. There seemed to be a direct correlation between how much I liked my partner and what my homework grades were. But I got an A++ on the first assignment, when I was working by myself. Perhaps because I felt I could rely on someone else to take up the slack and do the work for me (to a certain extent, anyway) I haven't done as well on the other homework.

JS: I enjoyed working with CN and had no particular problems,except that I felt that my background in mathematics made it impossible to contribute as much as her, and so I always felt a little guilty.

Question 8. If we had more resources, we could let everyone work alone on the problem sets. So we asked you to work in teams out of necessity. But some believe that it is beneficial for students to do the work in teams anyway. What do you think about that idea?

RB: Complete agreement. Suppose one of the students has too many things going on in other classes to get into the lab much one week -- his partner can cover for him and vice-versa if the other team member is too stressed. Having a partner also helps the learning process in that we learn better from our peers. Most concepts are much easier to understand when taught by a peer.

KC: No. It would actually be better if everybody worked alone and just helped each other out from computer to computer. There was more solo work this semester than last semester from what I saw.

SG: Personally, although it is beneficial to do some things in teams,it would probably be more beneficial to the student if he did it alone. He would not have to be dependent on the efforts of a partner, which, if the partner were not performing up to proper standards, could jeopardize the student's grade.

AH: I think everyone should have experience working in teams. But I would like to point out that teamwork is inevitable in an environment like C&M computer lab. Everyone helped everyone else. When I was stuck, I would often go to someone else on another team and ask questions or exchange ideas. Many times individuals would share information among themselves. No one had to tell them to do this, it happened naturally.

Often, teams would barter solutions to difficult problems to reduce their workload. I saw this happening frequently. I myself did it once.

So, my point is: in the C&M environment, teamwork is a necessary and inevitable way of life. No matter what -- it most assuredly will happen.

TM: I personally liked the idea and concept of working in teams on the problem sets. Although it was extremely frustrating at times, I realize that in the real world, you have to work in teams, whether you have good or bad partners. If possible in future classes an idea would be to let some students work alone if they preferred. That way some students would be in teams, where they prefer to be, and others who work better alone could go along at their own pace. To do this though, the makeup of the problem sets would need to be altered. This semester I found out the hard way that the problem sets are too long to be completed in one week by a lone student. The problem sets in the future should be scaled down a bit, or the students should be given longer to work on them. Another possible change that could be made with the problem sets, is their due dates. This semester problem sets were due on Wednesday nights. This posed severe complications when we had exams on Thursday mornings. We were always in the lab on Wednesday nights in a mad dash to complete the problem sets, and by the time they were completed we were too burned out to cram for the test the next morning. I realize that the 2/3 format of lecture/lab complicates changing this problem, but the outcome in the long run of solving this problem would outshine the difficulty in dealing with scheduling Exam and due dates.

CO: I think the team concept is very good because it teaches more than just haw to do math, but how to get things done with mutual cooperation.

MS: See the above answer. At least for me, working individually was the best choice.

JS: I absolutely agree, part of my problem with contemporary education is that it makes education a passive project of the individual subject. Team work, while required in the real world, is not allowed in the classroom. There are several advantages to the team set up. One of which is that team members discuss ideas among themselves before they enter them into the computer, this requires a certain amount of understanding by each team member before they can articulate their thoughts, the other team member then is able to augment the understanding of the first.

Question 9. For which of the material of Calculus II do you think the use of the computer was most helpful?

RB: Vectors, Parametrization, Polar Plotting, Interpolating Polynomials, Sequences and Series.

KC: All of it helped, especially the graphing which made it easier to see the ideas and functions accurately.

SG: In visualizing the graphs of conics, parametric equations, and vectors.

AH: I think that the true power of *Mathematica* reveals itself when students can approach a problem with a strong calculational and graphing tool at their disposal. I would say that the computer was useful for ALL the material we covered, especially vectors.

TM: The computer was extremely helpful in illustrating complicated 3D graphics and curves. The time and skill needed to draw comprehendible graphs could be devoted to actually learning the material instead of fussing over a poorly drawn plane in R3. The computer was also obviously helpful in executing problems that required much basic computation or algebra. There are certain problems in the material covered in Calculus II that the emphasis is on the result and not the lengthy, basic computations that allow you to get an answer. *Mathematica* allows you to focus on the theory and result of a problem without getting bogged down in the meaningless computations.

CO: It was very helpful for displaying complex graphs and more in-depth computations .

MS: By far the graphing, especially in three-space.

JS: I think all the work with integrals and polar graphing was made more accessible by use of the computer.

Question 10. For which of the topics in Math 1552 do you think the use of the computer was least helpful? Explain.

RB: Files 11-20 on Integration and techniques. It is so easy to let *Mathematica* do the integrals for you, that you don't try working them on paper for yourself first (blame it on American laziness).

KC: The computer was least helpful in repetitious statements and executions. Even if the work was tedious, it would have been simpler to just do it on paper to see what point was being made.

SG: In understanding conics, parametric equations, and vectors. I felt like I was just thrown into the material, and once thrown in, it was difficult to tread water and stay afloat.

AH: The computer was probably least useful when we studied integration by parts. There were not many problems presented to us that required the power of *Mathematica*.

TM: The computer was least helpful in discussing those areas where the 3D graphics, accurate graphing capability, and lightning fast computational work was not needed. As it sounds, there were not many areas in the course that couldn't be aided by these qualities. But I found the material on series and new integration techniques to be very difficult to understand. This type of material involved a few new calculation techniques that are best learned through practice, not illustration.

CO: It was also poor in explaining why the graphs were drawn the way they were.

MS: I cant think of any that it wasn't helpful.

JS: The Power series could have benefited from a more organized approach. I got more from reading a friends text on the power series than from working with the notebooks, but this could have been helped by more and better planned problems.

Question 11. Name two or three topics to which you devoted substantial study time but (a) with which you had difficulty or (b) which you disliked, and explain.

RB: Sequences and series. But then again, almost everyone hates those (students at least). I guess they're just not nearly as interesting as solving improper integrals with inverse trig functions (sarcasm).

KC: Polar graphs, series, and some vectors.They were easy to execute and learn the commands, but extremely difficult to do so on paper and were tricky also. They were all difficult.

SG: Conics, parametric equations, and vectors. Please see above explanation.

AH: I am really amazed by vector calculus! I think it is fascinating. But unfortunately, it was my downfall in this course. I had trouble with the subject and it killed my grade in the course.

I did not particularly care for the study of osculating circles. Will I ever use this??

And I have grown to dislike the word "parametric." Whenever I see this word I cringe and feel a deep sense of approaching doom.

TM: I was really stressed out about those hypocycloids and related material in the files! Thank God they were not covered in the exams in great detail. At that point in time in the course, we were bombarded with new material and those hypocycloids blew my mind.

CO: I had a great deal of trouble understanding the things involving the use and applications of vectors ,because they were not explained clearly in the material.

MS: I didn't like either the Taylor series or the Power series. Perhaps just because of my poor attendance during those topics, perhaps because I'm just stupid, I just couldn't seem to get it. Also curvature in 3-space. A total nightmare. I was there for the lectures, but the comprehension just never dawned on me.

JS: I disliked none, but found many trying. See above for the only legitimate complaint. Last semester I had more problems and I think this is due to the lectures and newness of the course.

Question 12. Name two or three topics which you found relatively easy, or which you particularly liked, and explain.

RB: I liked learning about the vectors, except that *Mathematica* uses a different notation than my physics and the Swokowski Calculus book use. I greatly prefer the new style on *Mathematica*, since we don't worry much with writing everything as an equation. This makes learning about 3-D vectors easier.

KC: The topics I found easy were integration. The ones I liked were interpolating polynomials and polar graphs. Integration is nothing but a command that's why it was so easy. Interpolating polynomials were fascinating because I had never thought that a graph could be produced from a set of points in an orderly fashion. Even though I said polar graphs were hard and time consuming, I had lots of fun playing around with the ways you could generate them and manipulate the curves. More than all though, I had the most fun doing parametrics with the 3D graphics. This was too awesome. I doubt anybody could produce graphics like *Mathematica* does by hand on a regular basis and make it so clear.

SG: Partial derivatives, integration by parts, and interpolating polynomials. These three ideas seemed to have the most "hands-on" experience for us, and just seemed to me personally to be the most fun.

AH: No easy topics. I liked vector calculus, but it was not easy. I keep thinking "I wish I had more time to approach these subjects. One semester is just not enough."

TM: I thought the material on Newton's method and the material dealing with circles and circles of curvature to be very beneficial. The material was presented in a straightforward way and was illustrated with relevant examples. The use of *Mathematica* was a great aid in removing much of the "meaningless computation",that I have described above at length.

CO: The portion dealing with integrals and area.

MS: Kepler's Laws. (I'm a Physics major.) Curvature in 2-space seemed pretty easy as well. Also integration of trig and inverse trig functions. We covered that (a little bit) in Calculus I as well (Professor Keisler).

JS: I liked them all but found none easy.

Question 13. On the whole, and considering its potential development, do you think that Calculus with *Mathematica* is a good idea? Why?

RB: Definitely. Especially with the higher level Math courses. The computers help to understand.

KC: It was a good idea. I would support it in any way I could. However, if it is to go on then the pencil-and-paper skills should be more emphasized or completely withdrawn, and hopefully not the latter. The only problem was that the work was being done by computer and the concepts by us which is completely fine on the computer, but when it came to a pencil-and-paper test then the pencil-and-paper skills were not very good. This could be a great developmental tool if the working of problems could also be demonstrated on computer.

SG: Yes, since it allows the student to visualize many of the concepts presented, but the computer should be a supplement to the material,not the major emphasis. There also seems to be more interaction between student and instructor, which is good.

AH: I think C&M is a great idea and it should be approached with vigor. But it needs to be redesigned. I don't feel it is as effective as it could become if it continues status quo.

TM: Yes. I think that programs like *Mathematica* will definitely revolutionize the way math, and soon, all subjects are taught. The computer is an invaluable resource, and should be utilized in the future for educational purposes. I only hope that emphasis on the human professor is not forgotten. The professor should always be a vital element in the course, not only as a lecturer, but also one to help with the inevitable computer glitches, and an overall role model as well.

CO: Yes, because it showed functions which would be impossible to see in a classroom.

MS: Yes. For the same reason I applaud the fact that the Computer Science department has finally added an entry-level C course to its curriculum. Part of college is learning just for learning's sake, but it also is supposed to prepare you for actually using the knowledge in the real world. Doing integration and graphing by hand, like the Pascal computer language, is rapidly going the way of the dinosaur in the real world.

JS: Yes, most definitely. I truly think that the kind of interaction that is required with the Mathematica notebooks goes a very great deal beyond the kind of passive reception of knowledge in a rote skills approach to education. If we ever hope to have a society where people can truly think about the ethical and social implications of their actions (not a bad idea in a democracy) then we must teach them to be active participants in learning.

I enjoyed this course and got a lot from it.

Question 14. Do you have any suggestions, in addition to any you may have written above, as to how the course could be improved?

KC: More computers. Better pencil-and-paper skills. More time to work on homework. More lab hours. Improved homework lessons (not so much work).

SG: There should be some way separate from the class itself to teach the student how to use Mathematica as it is meant to be used in the course.

AH: 1.) Get a better textbook to go along with the course.
2.) More lecture time.
3.) Lab assistants who are veterans of at least one war (i.e., those who have taken C&M at least one semester) staffing the lab at all times and pay them to help students, not just monitor the computers.
4.) Designate this as an honors course.
5.) Homework assignments handed out in class that assist students in developing good pencil and paper skills.

TM: I would like to see the text by Swokowski integrated more into the course. It is an excellent text and far outshines the $80 Illinois Materials.

Also, I would love to be of any help in improving the course for future classes with *Mathematica*.

If any work over the summer is done in this area I will be more than willing to offer time and suggestions. I will be in town all summer at my house.

CO: Have a better amount of explanation in the classes.

MS: Nope.

Question 15. What advice would you give to a student who is deciding whether to take a computer section of Calculus II?

KC: I'd tell them to take it, it's worthwhile. I would warn them about the pencil-and-paper skills that are not included in computerized homework that he does need. I'd encourage anyone who is willing to take this course to take it.

SG: Plan to spend a lot of time in Lockett 233 doing calculus and learning the system, and, if coming from a non-computer Calculus I or non-computer Calculus II, to keep the book from the previous course so you can use it to supplement the material.

AH: As Jesus told his disciples,"One that endeavors to build ... must first count the cost."

Likewise, I say to all of you who are considering taking C&M: "Count the cost. Understand you will work and will work hard. But there are rewards if you stick with it. A better understanding of Calculus in particular and mathematics in general, and a deeper appreciation of the power of the machine to help humans solve difficult problems. "

TM: I would tell them to schedule a good amount of time that they can devote to the course. This class is certainly not for everyone. But for those people who don't mind putting in a few hours a week and overcoming the initial frustration in learning the syntax of *Mathematica*, the benefits are certainly rewarding. The opportunity to use a program like *Mathematica* has definitely been an invaluable learning experience.

CO: I would recommend it because it allows you to be able to examine strange things.

MS: Take it. In fact, I've already given that advice to several of my friends for next semester.

Question 16. Any other comments?

KC: I think *Mathematica* is extraordinarily wonderful and it has helped me very much in learning concepts but not exactly how to do the work. It could be possible that one day an instructor might not be needed to teach a *Mathematica*-based course since most of it could be done through e-mail. Even tests could be administered this way with just a supervisor. The only ways to handle the overcrowding situations are to either install more computers, create more sections with the number of students equal to or less than the number of machines available, or do both. Both would definitely be the best thing to do. If you want to advertise *Mathematica*, I suggest that you increase the number of sections. Finally, keep the program.

SG: I have enjoyed taking this course. It has provided a new perspective for me on calculus, and has helped me to better visualize the material for Calculus II.. I wish that there had been more lecture sessions, as this would have allowed me to better understand the material as presented.

I do have one comment on the book for this course. The book does a very poor job of explaining the concepts that the student is expected to learn by hand. I think that a book like Swokowski or Thomas and Finney would provide the student with a better grasp of the material. The book needs to be rewritten in a form that the student may be better able to understand, and a separate list of the most-used commands should also be provided.

AH: Since I am a veteran of both wars, I cannot leave without saying that I have never had the privilege of meeting a finer man than Dr. O. Carruth McGehee. He is an exemplary individual and a fine professor. He, more that any other professor I have sat under in my four years in the LSU system, sets the example I believe all men in his position should project. He was always kind and considerate of the students. He never had a cross word for anyone. He graded fairly. He was very understanding of particular students' personal problems. I only wish that, when I achieve my dream of teaching in a university one day, I will exhibit the same good qualities to my students that he demonstrated to me.

TM: Overall as I look back at this survey, I think I may have been too harsh. It's just that I think this course has got great potential, so we need to get the few glitches out. I don't mean to "bash" anyone that helped in developing the material; I realize the amount of time that has gone into making this a great class. It's just that we are almost there in terms of making this the "perfect learning experience", so let's work hard to fine tune the class to its top potential.

MS: Nope. Only thanks for a great semester.

Appendix 2

A Sample of Student Opinions in Calculus I

In one of the two sections of Calculus I taught in Fall 1991 using *Mathematica,* the statements 13 through 20 below were added to the usual evaluation form. The instructions asked the student to respond to each statement with A,B,C,D, or E, with meanings as follows:

(A) Strongly agree
(B) Agree
(C) "On the Fence"
(D) Disagree
(E) Strongly Disagree

The table on the left gives the distribution of the responses.

A	B	C	D	E	
0	1	3	2	6	13. The Mathematica notebooks helped me to better understand the geometrical concepts of the Calculus.
0	3	4	1	4	14. Calculus and Mathematica has changed my view of the nature of mathematics.
2	1	2	4	2	15. Calculus with Mathematica is more interesting than the "standard" treatment of calculus.
0	2	5	2	3	16. Mathematica on the NeXT computer was easy to learn.
0	0	3	4	5	17. I have more confidence in my mathematical skills at the end of the semester.
0	1	4	2	5	18. I have a better understanding of mathematical ideas and concepts because of the assignments required written responses.
0	2	1	4	3	19. I am now better able to discuss verbally and to write about mathematical concepts.
2	1	3	2	4	20. Calculus with Mathematica has provided better tools for my future use of mathematics.

Appendix 3.

Written Comments from Four Students in Stoltzfus's Calculus I, Fall 1991

A: I think that *Mathematica* on the NeXT computer should be used as an aid to learning calculus. I think that there is too much emphasis on the computer. Working out problems by hand helps you learn the material, whereas when using the computer one simply learns the commands of the computer, rather than the material. I will take the traditional Calculus II out of the textbook, because I want to learn calculus. However, I do believe that the NeXT computer should be open to me to aid my learning in that class. This class was quite an experience! Thanks.

B: Having never taken calculus before I found the entire *Mathematica* approach very confusing. Teaching calculus to myself was not what I had in mind, before taking the course. Maybe, If I had taken calculus before I would gain a new understanding by taking it with *Mathematica*. But overall I would say that learning calculus like I would "baseball or tennis" just doesn't work.

E: I found this course to require much more time than I had expected. I did expect this to be a difficult course, but not as difficult as it turned out to be. I did not like the book. I found that I learned more using a conventional calculus book as a supplement to this course, than I would have just using the *Mathematica* book. A suggestion I have for this course is to explain the principles clearly and don't assume that the students understand. I realize that if a student is intimidated by a course they will be reluctant to ask questions. But seeing how the course is relatively difficult to begin with, the teacher should teach the material, not assume that the students understand.

F: This class could have been more enjoyable and interesting if the computer assignments were not so tedious and difficult. The text was unclear and near impossible to learn from without outside explanation. However, *Mathematica* has helped me in visualizing the mechanics of calculus. If a standard text was coupled with *Mathematica* and more time was spent learning the concepts the class might have been a success. I truthfully believe that some of my classmates are more confused about calculus now than before the class.