List 1 of Corrections and other Changes to
An Introduction to Complex Analysis
Those who have the second or later printing of the book can ignore this list.

Page xiv, line 20: Instead of "I will state and explain ...," let's just say: "In Section 2.4, I will invoke this formulation of it: Every nonempty set of real numbers which is bounded above has a least upper bound."

Page 17, line -8: 100^o, not 100^*.

Page 17, line -14: "b,d" should be "b,e."

Page 18, second paragraph, line 1: $p \in P$ and $P \ni p$.

Page 18: In the last display on the page, the fourth statement should be: $\emptyset \subset R^+$.

Page 25: In subsection C, paragraph 2, line 3, the words "function, one-to-one" should be deleted.

Page 28, line 3: minimum.

Page 48: The last two sentences of the proof should be: "Statement (iii) can be proved similarly, by taking the product of the three matrices. It also follows logically from statements (i) and (ii)."

Page 53, line 1: $B = \{z \mid |z| > 1\}$

Page 56: On lines 2 and 3 of Proposition 1.4.6, "B" should be "X."

Page 60, in item 1 under Help on Selected Exercises, the reminder should be expanded, thus: "Recall that $[p, q]$ denotes the line segment from p to q, and $[p_1, p_2, \cdots, p_n]$ the union of the line segments $[p_1, p_2], \cdots, [p_{n-1}, p_n]$." Also, item 1(b) should read as follows:

$$S_8^o = \{(x, y) \mid 0 < x < 2 \text{ and } 0 < y < 1\};$$

$$bS_8 = [(0,0), (2,0), (2,1), (0,1), (0,0)] \cup [(1,-1), (1,0)] \cup [(1,1), (1,3)].$$

Page 67, first line after Prop. 1.5.3: "For a proof, see [8, p. 194]."

Page 69, in the line just before display (11): C^2, not C^1.

Page 77, line 9: The second line of the display should be

$$+G_{uv}(2u_x v_x + 2u_y v_y).$$

Also, at the bottom of the page, let's reword Exercise 4 so that it doesn't mention i: "Let $u(x, y) = x^2 + 6xy - y^2$. Let v be a harmonic conjugate of u such that $v(0, 0) = 2$. Identify $v(x, y)$."

Page 80: The first display in Exercise 26 should begin "$u(x, y)$."

Page 81: The last line of Exercise 28 should read as follows:

$$u(x, y) := \frac{1}{(x^2 + y^2)^{1/2}} \quad ((x, y) \neq (0, 0)).$$

Page 84, lines 3-4 of subsection B: "... it seems best to write the definition first in terms of polar coordinates. In subsection 1.3B, we made this definition: ..."

Page 87, lines -11, -10: "where each coefficient a_k belongs to $\mathbf{C}$, with $a_n \neq 0$, factors completely over $\mathbf{C}$:

$$p(z) = a_n \prod_{k=0}^{n} (z - z_k) \ldots \text{"}$$

Page 95, line 1: The circle has center $-\overline{\alpha}/c \ldots$.

Page 96: My intention was not to use the notation e^{it} until after subsection 2.2B, where the exponential function is introduced. Thus on line 8 (counting from the beginning of subsection I) the equation should be "$z = p + (\!(r, \theta + \sigma)\!)$." In the display on the next line, the right-hand side should be "$p + (\!(r, \sigma - \theta)\!)$." On line 10 the right-hand side of the equation should be "$(\!(r, -\theta - \sigma)\!)$." And finally, in display (17) the right-hand side should be "$p + (\!(1, 2\sigma)\!)(\overline{z} - \overline{p})$."

Page 99: In Exercise 18, "$2e^{it}$" should be "$(\!(2, t)\!)$."

Page 102, at the very beginning of Section 2.2, it should be "1.1–1.3," not "I.1–I.3."

Page 103: The second sentence in the first bullet under "Powers of z" should be "The image of a line through the origin is another such line if n is odd, a half-line if n is even."

Page 105, line -5: A closing parenthesis is needed: (equivalently, $y = \arg w$).

Page 105, last line: Note that $\mathrm{Log}\, w = \ln|w| + i\mathrm{Arg} w$.

Page 106, in the display: "$2\pi in$" should be "$2\pi n$."

Page 117: The last sentence in the first paragraph should say "See, for example, the graph on the left in Figure 1.2-3 and its description in subsection 1.2C."

Page 124: The sentence that begins in line -7 should be: "Of the five examples in that figure, the only conformal linear mappings are those that take Grid A to the two grids in the last row."

Page 126, Exercise 1, last sentence: ... equal 0 and -7, respectively, ...

Page 129, lines 6 and 8, a font error: $\mathbf{S_n}$ should be in bold italic.

Page 132: For the sake of extra clarity, let's add "$\equiv \lim_{m\to\infty} b_m$" to display (4); and let's add "$\equiv \lim_{m\to\infty} a_m$" to display (5). And let's change the sentence after display (5) to "The right-hand sides of (4) and (5) are meaningful for every sequence $\{x_n\}$ of real numbers." Also, the rest of the page ignores the case when ℓ is not finite. To correct that, let's delete the sentence beginning with "Notice that" and change the proof of 2.4.3

to begin as follows: "We will give a proof only for the case when ℓ is finite. Notice that $\ell = \limsup_{n\to\infty} x_n$ if and only if for every $\epsilon > 0$,

$$\text{frequently } \ell - \epsilon < x_n \quad \text{and} \quad \text{eventually } x_n < \ell + \epsilon.$$

Applying this fact ..."

Page 137, Exercise 16, fourth line: Delete "is finite." In both Exercise 16 and 17, replace the last sentence by "Is the statement still true without the requirement that O be bounded?" Also, on line -4: Establish and use the compactness of $\overline{O}$.

Page 153, line 6: $|I(a_0, b_0, c_0)| \leq 3d^2 m_n$.

Page 154: Lines 16-19 (line 16 is the third display on the page) should read as follows:

$$\int_{t_0}^{t_1} \frac{w'(t)}{w(t) - z}\, dt = \log(w(t) - z)|_{w(t_0)}^{w(t_1)}.$$

Thus the integral's imaginary part equals $\arg(w(t_1) - z) - \arg(w(t_0) - z)$, which is the net change that takes place in $\arg(w(t) - z)$ as t increases from t_0 to t_1. That change is the same regardless of the choice of argument. ...

Page 159, next-to-last paragraph: "... is not $F = U + iV$ ..." (there should be only one "not.")

Page 161, first line: A **contour** Γ is the union ...

Page 164, line 2: Replace "Then" by "Prove that."

Page 173, lines 9, 10: The first two sentences in the proof of (iv) should read as follows: If $0 < r < R \leq \infty$, then $\limsup_{n\to\infty} |a_n r^n|^{1/n} = \dfrac{r}{R} < 1$. Choose s such that $\dfrac{r}{R} < s < 1$.

Page 181, last line: Delete "R of."

Page 182, Exercise 10(e): $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{4^n + 5^n} z^n$.

Page 183, Exercise 21: $\dfrac{z(1+z)}{(1-z)^3}$.

Page 188, display (2): The comma should be a period.

Page 193, Condition IV: ...every triangle in the disk with a vertex at z equals 0:

Page 196, in the display on line 16, the integral should be taken over the circle $|w| = s$, not $|w| = 1$.

Page 198, line 2: "3.1.8)." (The closing parenthesis was missing, that's all)

Page 203, item (p): Just for better appearance, let's say: "... one-to-one onto the set of all $z = x + iy$ such that $|z| = 1$, $x \geq 0$, and $y \geq 0$.

Page 203, Help on Selected Exercises: The answer to 1(f) should be $-4i$. The answer to 2(b) should be $10\pi i$.

Page 207: In Figure 3.2-2, one of the little arrows that points to the right should point to the left instead. It's the one that's about two inches above the words "but with" in the caption.

Page 211, Exercise 1: ... complement of O?" (The quotation marks were missing). Also, in Exercise 7, a font error: "Log" should not be italicized.

Page 212, item 7, line 2: "*Hints:* Note that $|n^z| = |e^{z \ln n}| = e^{x \ln n} = n^x$ if $z = x + iy$. Let $x_0 > 1$. Then

$$\sum_{n=m}^{\infty} n^{-x} \to 0 \quad \text{as } m \to \infty, \text{ uniformly for } x \geq x_0.$$

Thus the series for ζ converges subuniformly on O."

Page 216, in Remark 3.3.4: ...the cotangent, given by $\cot z := \dfrac{\cos z}{\sin z}$,...

Page 221, display (17): The denominator should be $(-2i)^k$ instead of $(2i)^k$.

Page 228, line 11: It should be "$R \to \infty$."

Page 235, line 6: "product," not "sum." Page 238: 3.4.2(iv) and 3.4.2(v) should both begin with: "Let f be meromorphic at $p \in \mathbf{C}$."

Page 241, line 11: It would be best not to risk leaving the impression that the residue is zero whenever the singularity is removable. So let's revise this line to say "The singularity of f at ∞ is removable, and $\operatorname{Res}(f, \infty) = 0$, by 3.3.22(ii). By 3.3.15,"

Page 246, In the first display, Ω should be O.

Page 247: The first two lines should say: "It follows that if g is one-to-one and $m(g, z) \equiv 1$ on V, then there is an integral representation for the inverse function $(g|V)^{-1}$:" Note: The condition $m(g, z) \equiv 1$ (equivalently: g' has no zeros) is automatic from the one-to-oneness of g, since if $g'(z) = 0$, then g is at least two-to-one on some punctured disk at z; this is proved in Section 3.5. Also, on line 8: "... on which g is one-to-one."

Page 250, Exercise 21: Insert the factor $\dfrac{1}{2\pi i}$ in front of each of the nine integrals.

Page 251, Help, item 17: The integral equals ... $\dfrac{\pi i(e^2 - 1)}{e}$ for $r > 1$.

Page 251, Help, item 21(e): $2e^{12}$. Item 21(g): 17. Item 21(i): -36.

Page 252, line -5: "it is," not "is is."

Page 256, line -2: ... g is continuous on S.

Page 262: The line defining the stereographic projection should be:

$$\tau^{-1}(\xi, \eta, \zeta) = \left(\frac{\xi}{1 - \zeta}, \frac{\eta}{1 - \zeta}, 0 \right).$$

Page 270, Exercise 1: "Let $a > 1$."

Page 271, Exercise 10: Delete the "4." The value should be $\dfrac{\pi(a^2+b^2)}{a^3b^3}$.

Page 274, line -8: The first line of the big display should begin with the modulus of that integral:

$$\left|\int_{\mathrm{Arc}(R,0,\frac{\pi}{4})} e^{-z^2}\,dz\right|$$

Also, in the second line of the big display, "=" should be "$\leq$."

Page 280, lines 4-5: ...Consider now the pullback of the third integral, using the parametrization $x \mapsto xe^{i\alpha}$ $(R \geq x \geq 0)$: ... Also, in the second and third displays on this page, the left-hand sides are incorrect. The second display should begin with $\displaystyle\int_{[Re^{i\alpha},0]} f$, and the third with $\displaystyle\int_{[Re^{i2\pi/3},0]} f$.

Page 288, in the solution to Example 4.4.6: Let's say "Because the integrands have infinite limits ...."

Page 290, line 7 of the Solution: "β_R" should be "$\mathrm{Arc}(R,0,\frac{\pi}{4})$."

Pages 308 and 315: "Hints for Selected Exercises" should be "Help on Selected Exercises."

Page 312, line -5: The denominator should be $(R - |s-\alpha|)$, not $(R - |s|)$.

Page 315, line -7: (c) $e^{-t}\cos \pi t$.

Page 321, line -5: There is an incorrect sign in the display. It should be

$$v(z) = \frac{50}{\pi}\ln|z-1| - \frac{90}{\pi}\ln|z-2|.$$

Page 326, Exercise 4: "65^o," not "85^o."

Page 329, line -10: If it has exactly two fixed points, it is **loxodromic**.

Page 333: In Figure 5.2-1, there appear four circles through -1 and 1. Their images are the four lines through the origin, other than the real axis, shown in Figure 5.2-2. Each circle and its image-line are marked with the same symbol. The part of each circle that lies in the second quadrant is mapped to the part of the line that lies inside the upper half of the unit disk. Therefore it would be better if the symbol on each of the lines were placed within distance 1 of the origin. Also, the caption of Figure 5.2-2 should refer not to Figure 5.2-2, but to Figure 5.2-1.

Page 340, Exercise 29: the proof of 5.2.21.

Page 350, Exercise 4: The potential graphs in Figures 5.3-1 and 5.3-2 carry ...

Page 352, line 4: "... that σ is bounded at infinity."

Page 353: The first display should be

$$g(re^{it}) = \frac{1}{2}\left(re^{it} + \frac{1}{r}e^{-it}\right).$$

In the second display, "$+\frac{1}{2}$" should be "$+\frac{i}{2}$." Also, in equation (2) at the bottom of the page, the factors of 2 in the denominators should be deleted.

Page 361, display (16): "dt" should be "$d\theta$."

Page 364, second display: "dt" should be "$d\theta$."

Page 372, line -14: "... Taylor series at a_0,

$$G(z) = \sum_{k=0}^{\infty} c_k(z - a_0)^k,$$

are all real." Also, on the second line of Proposition 5.7.2: "... let F be the holomorphic extension of f to $O \cup O^*$ such that $F(z) = \overline{f(\overline{z})}$ for $z \in O^*$." This extension is given by (1) on page 370.

Page 373, on the third line of Theorem 5.7.3: "... let G be the holomorphic extension of g to $U \cup U^*$ such that $G = J_B \circ g \circ J_A$ on U^*."

Page 381, line 13: "$-ie^{\pi/c}$" should be "$-i\epsilon^{\pi/c}$."

Page 383, bottom line: The integral should run from 0 to z, not 0 to 1.

Page 404: In the statement of the Theorem: Let O be a non-empty, simply connected, ... In the proof, the function h is mis-defined; it should be $1/(z-c)$ for suitable c. Instead of just fixing that error, I'm offering a revision of the entire portion of the proof that lies on page 404, thus: *Proof.* The mapping $z \mapsto z - a$ is a conformal equivalence of the sets O and

$$O - a := \{z - a \mid z \in O\}.$$

Therefore, when we set out to prove that O is conformally equivalent to another set, we are free to assume that $0 \in O$; or to assume that $0 \notin O$.

We are also free to assume that O is bounded, because there is always a holomorphic equivalence from O onto a bounded simply connected open set. That claim can be proved as follows. We may assume that $0 \notin O$. By 3.2.3, page 208, there exists a holomorphic function g defined on O such that

$$g(z)^2 = z \quad \text{for all} \quad z \in O.$$

Notice that if $z_1, z_2 \in O$ and $g(z_1) = \pm g(z_2)$, then $z_1 = z_2$. Therefore g is one-to-one; also, if $w \in g[O]$, then $-w \notin g[O]$. Since $z \mapsto -g(z)$ is an open mapping, there is a disk $D(c, r)$ contained in $\{-w \mid w \in g[O]\}$. The function $h : z \mapsto \dfrac{1}{z - c}$ maps $D(c, r)$

onto $\{z \mid |z| > 1/r\}$, a neighborhood of ∞. Evidently $D(c,r) \cap g[O] = \emptyset$, and therefore $h[g[O]] \subset D(0, 1/r)$. Thus the mapping

$$z \mapsto h(g(z)) \equiv \frac{1}{\sqrt{z} - c}$$

is a conformal equivalence from O onto a bounded, simply connected, open set. Note the use here of Proposition 3.2.5.

Page 405, line 7: ... is subuniformly equicontinuous on O.

Pages 408-409: Starting at the last line of page 408 and in the rest of the Lemma's proof, each time the symbol s appears, it should be replaced by t.

Page 409, line 3: The c should be b.

Page 409, line 14: $[0, f(z(x_j))]$.

Page 409, line 16: $[f(z(y_j)), 0]$.

Page 409, line -12: $|f(z(t))| > |z_0| \cdots$.