ORTHOGONAL REPRESENTATIONS: FROM GROUPS TO HOPF ALGEBRAS

In recent years Hopf algebras have had applications in mathematical physics, in particular conformal field theory, and in other parts of mathematics, such as knot theory and operator algebras. In this talk we discuss some recent progress in the representation theory of Hopf algebras which extends classical work in group theory.

Frobenius and Schur showed in 1906 that one can decide whether or not a complex representation $V$ of a finite group $G$ is real by computing the value of

$$\nu(V) = \frac{1}{|G|} \sum_{g} \chi(g^2),$$

where $\chi$ is the character of $V$.

$\nu(V)$, the indicator of $V$, takes only three values, 0, 1, or -1. The representation is real precisely when $\nu(V) = +1$; equivalently $V$ has a symmetric non-degenerate $\mathbb{C}$-bilinear $G$-invariant form. Thus the elements of $G$ act as orthogonal transformations on $V$.

In the last decade, Frobenius-Schur indicators have been extended to finite dimensional Hopf algebras and beyond, such as to quasi-Hopf algebras and fusion categories; they are invariants of the monoidal (tensor) category of representations. There have been many recent applications of indicators, such as results about classification or about the exponent of the Hopf algebra.