SOBOLEV MAPPINGS AND ENERGY INTEGRALS
IN NONLINEAR ELASTICITY
Colloquium at LSU, November 2013 (VIGRE lecture)

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1. Abstract

Throughout this talk $X$ and $Y$ are bounded domains in $\mathbb{R}^n$. Although Riemann surfaces and $n$-manifolds are not in the center of our discussion, the ideas really crystalize in a differential-geometric setting. Thus we suggest to think of $X$ and $Y$ as Riemannian $n$-manifolds. The topics are about functions, called Sobolev mappings

$$h : X \rightarrow Y, \quad h \in W^{1,p}_{\text{loc}}(X, \mathbb{R}^n), \quad 1 \leq p \leq \infty$$

We adopt various interpretations from the theory of Nonlinear Elasticity (NE), where a great part of our discussion is highly motivated. Our discussion is about the mapping problem in Geometric Function Theory (GFT) versus hyperelastic deformations in Materials Science. Both theories share compelling interest through the energy-minimal solutions of Variational Integrals. In quest of smallest energy we discover the collapsing phenomenon in the minimization of the Dirichlet integral (interpenetration of matter occurs). Theoretical prediction of failure of bodies caused by interpenetration of matter is a good motivation that should appeal to mathematical analysts and researchers in the engineering fields. As we seek greater knowledge about the energy-minimal deformations (not necessarily injective) the questions about their existence and regularity, as well as the general Sobolev homeomorphisms and their limits (monotone Sobolev mappings), become ever more quintessential.

Graduate students are encouraged to come. They are especially welcome.

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