Math 7390, Section 1
Harmonic Analysis I, Fourier Analysis and Distributions

Textbook: Lecture notes by R. Fabec and G. Ólafsson. Those notes are available at https://www.math.lsu.edu/harmonic/
Time: 9:40-10:30, Monday, Wednesday and Friday in Lockett 111)
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web-page: www.math.lsu.edu/∼olafsson. This syllabus, list of problems, test dates, and solutions to tests, quizzes and other information will be available on this web-page.

SYLLABUS

The subject proper of *harmonic analysis* is to decompose functions into "simpler" functions, where the meaning of *simple* depends on the context we are working in. In the theory of differential equations that means to write an arbitrary functions as a superposition of eigenfunctions. If we have a symmetry group acting on the system, then we would like to write an arbitary function as a sum or superposition of functions that transforms in a simple and controllable way under the symmetry group. The simplest example is the use of polar coordinates and radial functions for rotation symmetric equations. Sometimes, we are working with general spaces than $\mathbb{R}^n$. Here anlysis meets with Lie groups, geometry, representation theory and harmonic analysis to form *abstract harmonic analysis*.

The course begins with a short overview of classical Fourier analysis on the torus and $\mathbb{R}^n$. This leads us to topics like:

- Periodic functions and Fourier series;
- Convergence of Fourier series;
- Spaces of functions on $\mathbb{R}^n$. In particular, we will discuss the space of compactly supported functions, functions of compact support and the algebraic structure of those spaces, i.e., convolution.
- The Fourier transform of rapidly decreasing functions and L^2functions, inversion formula and Plancherel theorems.
- Introduction to distribution theory and the continuous linear functionals on function spaces. How to differentiate distributions. The Fourier transform of distributions.

- Application of the Fourier transform to differential equations. In particular we will discuss the heat equation and the wave equation.
- Hermite functions and polynomials.
- At the end, we will also discuss some other integral transforms. In partiuclar, we will discuss the continuous wavelet transform, derive a Plancherel formula and an inversion formula.

We can view $\mathbb{R}^n$ as a set or as amanifold. But we can also view it as an *abelian group*. In that sense $\mathbb{R}^n$ is a part of *abelian harmonic analysis*. The simplest example of *nonabelian harmonic analysis* is the Heisenberg group $H_n = \mathbb{R}^{2n+1}$ (with a new group multiplication). The Heisenberg group is also a simple example of a *Lie group* and of a *topological group*. There are several other well known examples of topological groups like the group of rotations $\mathrm{SO}(n)$, the group of all invertible matrices $\mathrm{GL}(n, \mathbb{R})$. Depending on the time and interest we will at the end discuss some advanced topics related to topological groups.

The second part of this class takes plase in the spring 2008, where the weight is on topological groups, homogeneous spaces and representation theory. The instructore will be Mark Davidson

REFERENCES

- The Lecture Notes: **Noncommutative Harmonic Analysis** by R. Fabec and G. Olafsson;
- **Fourier Analysis** by T. W. Koerner;
- **Noncommutative Harminic Analysis** by Michael E. Taylor;
- **Harmonic Analysis on Phase Space** by G. Folland

HOMEWORK AND EXAMS

There will be the following requirements:

- Set of homeworks every second week;
- A midterm exam;
- Instead of a final, the students will have to give presentation on a selected topic in harmonic analysis. As an example:
  - The Fourier transform and the heat equation;
  - The wave equation in two and three dimensional space; Why can we listen to music in three dimensional space and not in a two dimensional space;
  - Behavior of Fourier series at points of discontinuity;

  just to name few possibilities.