Solutions

MATH 2025 Fall 2009

Assignment 1

Due Thursday, September 10, before the class

For full credit, show all your work!

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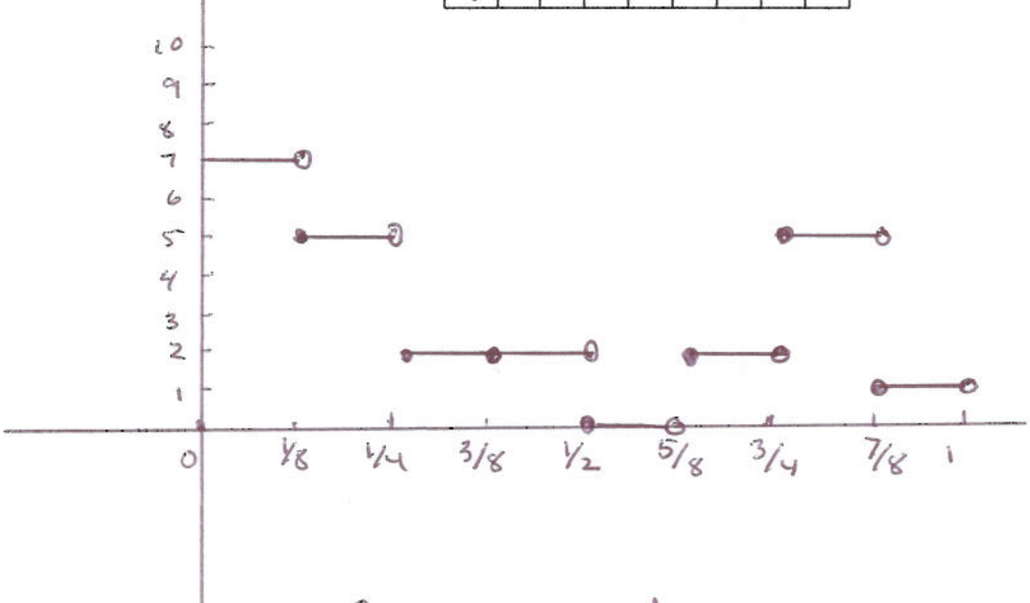
1. Plot and write a formula for the step function $\tilde{q}$ corresponding to the sample in the following table 5 pts

j	0	1	2	3	4	5	6	7
r_j	0	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$
s_j	7	5	2	2	0	2	5	1



Both ways to write $\tilde{q}$ are correct

$$\tilde{q} = 7\varphi_0^{(3)} + 5\varphi_1^{(3)} + 2\varphi_2^{(3)} + 2\varphi_3^{(3)} + 2\varphi_5^{(3)} + 5\varphi_6^{(3)} + \varphi_7^{(3)}$$

$$= 7\varphi_{[0,\frac{1}{8})} + 5\varphi_{[\frac{1}{8},\frac{1}{4})} + 2\varphi_{[\frac{1}{4},\frac{3}{8})} + 2\varphi_{[\frac{3}{8},\frac{1}{2})}$$

$$+ 2\varphi_{[\frac{5}{8},\frac{3}{4})} + 5\varphi_{[\frac{3}{4},\frac{7}{8})} + \varphi_{[\frac{7}{8},1)}$$

You can also replace $2\varphi_{[\frac{1}{4},\frac{3}{8})} + 2\varphi_{[\frac{3}{8},\frac{1}{2})}$ by $2\varphi_{[\frac{1}{4},\frac{1}{2})}$.

2. Calculate the *ordered Fast Haar Wavelet Transform* for the data $\mathbf{s} = (7, 5, 2, 2, 0, 2, 5, 1)$. 9 pts

Solution:

Initialization: $\vec{a}^{(3)} = (7,5,2,2,0,2,5,1)$

1^{th} step:

$\vec{s}^{(3)} = \vec{a}^{(3)}$

$\vec{a}^{(2)} = (6,2,1,3)$

$\vec{c}^{(2)} = (1,0,-1,2)$

$\vec{s}^{(3-1)} = (6,2,1,3;1,0,-1,2).$

2^{th} step:

$\vec{a}^{(2)}$

$\vec{a}^{(1)} = (4,2)$

$\vec{c}^{(1)} = (2,-1)$

$\vec{s}^{(3-2)} = \vec{s}^{(1)} = (4,2;2,-1;1,0,-1,2).$

Final step:

$\vec{a}^{(1)}$

$\vec{a}^{(0)} = (3)$

$\vec{c}^{(0)} = (1)$

$\vec{s}^{(3-3)} = \vec{s}^{(0)} =$

$= (3;1;2,-1;1,0,-1,2)$

Final answer

$$\boxed{\vec{s}^{(3-3)} = (3;1;2,-1;1,0,-1,2)}$$

3. Write the results in the **first** and **second** step in problem 2 as a combination of the functions ϕ and ψ. 7 pts

$$\tilde{f} = 7\varphi_6^{(3)} + 5\varphi_1^{(3)} + 2\varphi_2^{(3)} + 2\varphi_3^{(3)} + 2\varphi_5^{(3)} + 5\varphi_6^{(3)} + \varphi_7^{(3)} \quad \text{(not required)}$$

$$= 6\varphi_0^{(2)} + 2\varphi_1^{(2)} + \varphi_2^{(2)} + 3\varphi_3^{(2)}$$
$$+ \psi_0^{(2)} - \psi_2^{(2)} + 2\psi_3^{(2)}$$

$$= 4\varphi_0^{(1)} + 2\varphi_1^{(1)}$$
$$+ 2\psi_0^{(1)} - \psi_1^{(1)}$$
$$+ \psi_0^{(2)} - \psi_2^{(2)} + 2\psi_3^{(2)}$$

(You can also use the notation

$\varphi_1^{(3)}(r) = \varphi_{[\frac{1}{8}, \frac{1}{4})}(r) = \varphi(2^3 r - 1)$ etc.)

4. Assume that the *ordered Haar Wavelet Transform* of a sample $\mathbf{s} = (s_0, s_1, s_2, s_3)$ produces the results $\mathbf{a}^{(2-2)} = (6)$, $\mathbf{c}^{(2-2)} = (1)$, and $\mathbf{c}^{(2-1)} = (2, 2)$. 9 pts

(a) Explain how $a_0^{(2-2)} = 6$ relates to the sample;
(b) Explain how $c_0^{(2-2)} = 1$ relates to the sample;
(c) Explain how $c_0^{(2-1)} = 2$ relates to the sample.

Solution:

(a) The number $a_0^{(2-2)} = 6$ is the average of the samples:

$$6 = \frac{s_0 + s_1 + s_2 + s_3}{4}.$$

(b) $c_0^{(2-2)} = 1$ means that at the midle of the sequence (or sample) there is a jump as we go from the average of the first two numbers to the average of the last two numbers. This jump is 2.

[There are other ways to describe this. One can evaluate

$$\frac{s_0 + s_1}{2} = 6 + 1 = 7 \quad \text{and} \quad \frac{s_2 + s_3}{2} = 6 - 1 = 5$$

to name another example. See also the discussion in the book on page 17.]

(c) $c_0^{(2-1)} = 2$ says that $\frac{s_0 - s_1}{2} = 2$. You can also explain this in words.